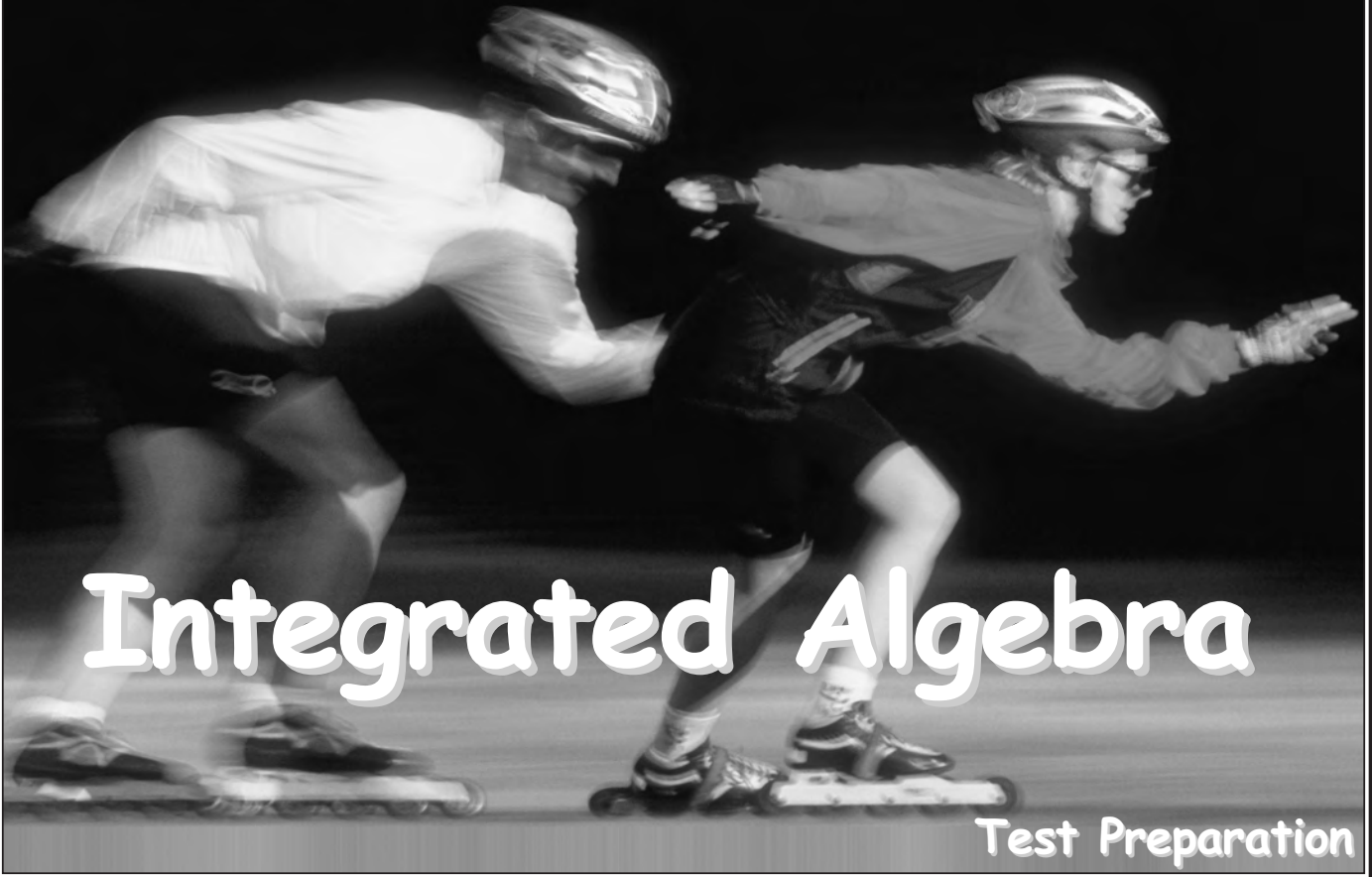


X-treme Review



Integrated Algebra

Test Preparation

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DEDICATION

This book is dedicated to our families and friends who supported us through this challenging process.

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We thank all of you from the bottom of our hearts.

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TO THE STUDENT

This *Integrated Algebra X-treme Review* is designed to help you achieve success on the state Integrated Algebra assessments. Each chapter contains:

Vocabulary

These words and phrases help you identify ideas and operations in Math and are often used as correct responses to questions throughout the lesson.

Specific Topics

Specific Topics are the “Strands,” also called the Key (Main) Ideas. They are to be learned in preparation for the Integrated Algebra Test.

Practice

Each Lesson has many practice questions. They are based on the “Bands,” also called the “Performance Indicators.” You must be able to understand and do them for success on the Integrated Algebra Test.



X-treme Notes, Hints, and Remember

These special “Notes” and “Remember” hints give you pointers and mental aids to remember the main ideas used on the Integrated Algebra Test.

Test Preps

These questions are similar to the questions you will have to answer correctly on the Integrated Algebra Test.

Two Complete Tests

These practice tests give you the opportunity to rehearse with questions on the level of the Assessments in order to do well on the Integrated Algebra Test.

Tips for Taking the Test and References

This tells you what tools and notes you can and cannot have with you during the test.

Make Your Study *X-treme!* Good luck!

COMMON MATH SYMBOLS

When taking the Integrated Algebra Test, the student is expected to recognize some common math symbols found in the questions and graphics. Students are also expected to use some common math symbols when answering of questions and showing work. Review the symbols in the chart.

SYMBOL	MEANING
+	positive, plus, or add
-	negative, minus, or subtract
±	plus or minus
× or •	multiply or times
÷ or /	divide
=	equals
≠	does not equal
≈	approximately equals
~	similar to
≡	congruent to
>	greater than
<	less than
≥	greater than or equal to
≤	less than or equal to
∴	therefore
△	triangle
π	pi or 22/7 th or 3.1415926535
⊥	perpendicular to ...
	parallel to ...

SYMBOL	MEANING
	absolute value bars
∈	an element of ...
∉	not an element of ...
∪	the set of ...
∩	the intersection of ...
⊆	a subset of ...
!	factorial
()	parentheses (grouping) does not include end number(s)
[]	brackets (grouping), includes end number
{ }	braces (grouping), represent a set
∞	infinity
∑	sum of ...
↔ AB	line AB
\overline{AB}	segment AB
\overrightarrow{AB}	ray AB
°	degree(s)
∠	angle
√	square root

LESSON SIX

FACTORING MONOMIALS AND POLYNOMIALS



A “FALLING” SENSATION

Skydivers generally do not experience a “falling” sensation due to the fact that the resistance of the air to their body at speeds above about 50 mph provides some feeling of weight and direction. At normal exit speeds for aircraft (approx. 90 mph) there is little feeling of falling just after exit, but jumping from a balloon or helicopter can create this sensation. They reach terminal velocity around 120 mph (190 km/h) for belly to Earth orientations, 150-200 mph (240-320 km/h) for head down orientations and are no longer accelerating towards the ground. At this point the sensation is like being in a strong wind. When they leave the plane, their momentum from the plane causes their direction of travel to change from the direction of the airplane’s flight (horizontal) to the direction pulled by the force of gravity (vertical). Skydivers call this transition period “the

hill,” and the amount of distance they fly with the plane due to the momentum is called “forward throw.” For typical people, less than 1g of force along the body’s long axis is what causes the “stomach in your throat” feeling on a roller-coaster or other amusement park rides.

Most skydivers make their first jump with an experienced and trained instructor (this type of skydive may be in the form of a tandem skydive). During the tandem jump, the jumpmaster is responsible for the stable exit, maintaining a proper stable freefall position, and activating and controlling the parachute. With training and experience, the fear of the first few jumps is supplanted by the tact of controlling fear so that one may come to experience the satisfaction of mastering aerial skills and performing increasingly complicated maneuvers in the sky with friends.



COMPUTING SKY DIVING

Did you know that skydiving is dependent on knowing about polynomials? The formula for a freefall is

$$h = .5(-9.8\text{m/sec}^2)t^2 + vt + s$$

(-9.8m/sec^2) is acceleration due to gravity. The t is time in seconds, the v is initial velocity (which can be assumed to be 0 because of leaving from a plane) and the s is the initial starting (jumping) altitude which is approximately 5,500 feet. When a person jumps from the plane, the parachute, which creates air resistance, acts against this formula. It can be stated that the parachute increases the time of the fall and decreases

the acceleration. This is why a skydiver lands safely. In this lesson on polynomials, you will learn how to do operations that are essential to understanding real life examples such as this one.

Story & Graphic Source: <http://en.wikipedia.org/wiki/Parachuting>
http://www.utdanacenter.org/mathtoolkit/downloads/alg1assess/alg1_9_diving.pdf

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LESSON 6 – FACTORING MONOMIALS AND POLYNOMIALS


VOCABULARY

These words and phrases are associated with factoring monomials and polynomials and may be used when answering questions in this chapter. Definitions and explanations can be found in the Glossary/Index at the back of this *X-treme Review*.

binomial	factor	linear term	polynomial
conjugate	FOIL	monomial	trinomial
decreasing degree	greatest common factor	perfect squares	


6.1 ADDING AND SUBTRACTING MONOMIALS AND POLYNOMIALS

An expression with one term is called a(n) (1) _____ . An expression with many terms is called a(n) (2) _____ .

 **Remember:** Addition and subtraction of monomials is the same as combining like terms. Monomials can only be combined that have the same letter or letters raised to the same power.

Example A: $-5x$ and x can be combined to form $-4x$. $6x^2y$ can be combined with $5x^2y$ to form $11x^2y$.

However, $-x^2$ cannot be combined with $5x$, and $-x^2y$ cannot be combined with xy^2 .

 **Remember:** To add or subtract polynomials, combine the terms or monomials in each polynomial that are alike.

Example B: Find the sum of : $(3x^2 - 5x + 9)$ and $(x^2 - 10 - x)$

$$\begin{aligned} (3x^2 + x^2) + (-5x + -x) + (9 + -10) \\ (4x^2) + (-6x) + (-1) \\ 4x^2 - 6x - 1 \end{aligned}$$


Hint: Use the associative property to group like terms.

Note: The answer is arranged in (3) _____ which means that the exponents in each term go downward. The first term has a degree of 2, the second term has a degree of 1, and the last term has a degree of 0.

When finding a sum, line up the like terms vertically as follows:

Example C: $(-8x^2 - 5x - 9) + (-3x^2 - 5)$

$$\begin{array}{r} -8x^2 - 5x - 9 \\ -3x^2 \quad -5 \\ \hline -11x^2 - 5x - 14 \end{array}$$

 **Remember:** When subtracting, the signs on all monomials within the second polynomial must change.

Example D: From $x^2 - 4x$, subtract $3x^2 - 2x + 10$

Hint: Whatever expression follows from goes first.

$$\begin{aligned} (x^2 - 4x) - (3x^2 - 2x + 10) \\ x^2 - 4x - 3x^2 + 2x - 10 \\ -2x^2 - 2x - 10 \end{aligned}$$

Note: All signs in the 2nd polynomial are changed.

It is important to remember that if subtraction is done vertically, all signs still change for the second polynomial

Example E: Subtract:

$$\begin{array}{r} -3x^2 - 7x + 4 \\ 5x^2 - 3x - 2 \\ \hline \end{array}$$

Becomes $-3x^2 - 7x + 4$

$$\begin{array}{r} -5x^2 + 3x + 2 \\ -8x^2 - 4x + 6 \end{array}$$

Note: Signs change for 2nd polynomial.



Remember: When a question asks to “combine like terms,” “simplify the expression,” or “add and subtract polynomials,” the same thing is being required.

PRACTICE

Directions: Answer each of the following.

- 1 Subtract: $-8mn^2$ from mn^2 _____
- 2 Combine like terms: $3x^3 - 6x + 5x^2 - 4x + 7$ _____
- 3 Simplify: $(5x^2 - x - 1) + (-4x^2 - 3x - 3)$ _____
- 4 Simplify: $(3x^2 - 4x) + (x^3 - 3x^2 + 8) + (-4x + 2)$ _____
- 5 Simplify: $(7x^2 - 5x - 3) - (x^2 - 2x - 1)$ _____
- 6 Subtract: $\begin{array}{r} 6x^2 - 2x - 3 \\ 8x^2 - x + 9 \\ \hline \end{array}$ _____
- 7 Find the sum of $(6x^2)$ and $(-10x^2)$ _____
- 8 Find the sum of $(-6x^3 + 8x - 9)$ and $(-3x^2 - 12)$ _____
- 9 Subtract $(-3x^2 + 7x)$ from $(x^2 - x - 9)$ _____
- 10 From the sum of $(-x^2 - 4x - 1)$ and $(-5x - 1)$, subtract $(-2x^2 - 3x)$ _____

6.2 MULTIPLYING MONOMIALS AND POLYNOMIALS

When multiplying and when the bases are the same, add the exponents: $x^a \cdot x^b = x^{a+b}$



Note: When multiplying monomials, apply the same above rule and apply the following steps:

- 1) Multiply the coefficients.
- 2) Multiply like variables by adding the exponents.

Example A:

1)	$(-3x^3)(-4x^2)$ $-3 \cdot -4 = 12$	Multiply the coefficients.
2)	$x^3 \cdot x^2 = x^{3+2} = x^5$ $= 12x^5$	Multiply the variables. Combine both products.

Example B:

$$\begin{aligned} &(-2y^3x)(4x^4y) \\ &(-2 \cdot 4)(x \cdot x^4)(y^3y) \\ &-8x^5y^4 \end{aligned}$$

Example C: If you are told to raise $(-3x^2y)^3$, do the following:

$$(-3x^2y)(-3x^2y)(-3x^2y) = -27x^6y^3$$

Raise the coefficient to the third power $(-3)^3 = -27$ and multiply the powers $(x^2)^3 = x^6$ and $(y)^3 = y^3$



Note: This is really taking a power of a power. However, a faster way of doing this example is to follow the formula: $(x^a)^b = x^{ab}$

Example D:

$$\begin{aligned} &(-5x^3y^2)^4 \\ &(-5)^4(x^3)^4(y^2)^4 \\ &625x^{12}y^8 \end{aligned}$$



Remember: When multiplying a monomial by a polynomial, apply the distributive property.

Example E:

$$\begin{aligned} &7x(-x^2 + 3x - 7) \\ &7x(-x^2) + 7x(3x) + 7x(-7) \\ &-7x^3 + 21x^2 - 49x \end{aligned}$$

Distribute the 7x and multiply.



Note: Sometimes after multiplying, it is necessary to combine like terms.

Example F: Simplify the following expression:

$$\begin{aligned} &-3x(x - 4) - 2(x + 9) \\ &-3x^2 + 12x - 2x - 18 \\ &-3x^2 + 10x - 18 \end{aligned}$$

Distribute.
Combine like terms.

A polynomial with two terms is called a(n) (1) binomial. When multiplying two binomials, you often get a polynomial with three terms that is called a(n) (2) trinomial.

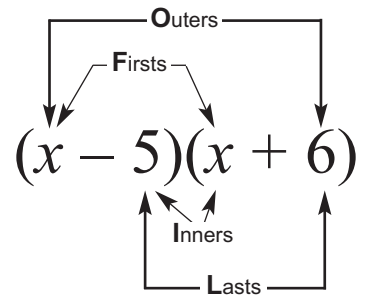


Remember: To multiply two binomials, distribute each of the terms in the first binomial to each of the terms in the second binomial or **FOIL (First, Outers, Inners, and Lasts)** and see illustration below.

Example G – FOIL Method: $(x - 5)(x + 6)$

- | | | |
|------------------------------|--------------------|----------------------|
| 1) Multiply the first terms. | $x \cdot x = x^2$ | x^2 |
| 2) Multiply the outer terms. | $x \cdot 6 = 6x$ | $x^2 + 6x$ |
| 3) Multiply the inner terms. | $-5 \cdot x = -5x$ | $x^2 + 6x - 5x$ |
| 4) Multiply the last terms. | $-5 \cdot 6 = -30$ | $x^2 + 6x - 5x - 30$ |
| 5) Combine like terms. | $6x - 5x = x$ | $x^2 + x - 30$ |

Answer: $x^2 + x - 30$



Example H – Distribution Method: $(3x - 2)(x + 7)$

- | | | |
|---|-------------------|-------------------|
| 1) Distribute $3x$ over $(x + 7)$. | $3x(x + 7)$ | $3x^2 + 21x$ |
| 2) Distribute the -2 over $(x + 7)$. | $-2(x + 7)$ | $-2x - 14$ |
| 3) Combine like terms. | $-2x + 21x = 19x$ | $3x^2 + 19x - 14$ |

Answer: $3x^2 + 19x - 14$

Note: Distribution also can be used to multiply trinomials.



Remember: If two binomials are alike, but one is a sum and the other a difference (can be called conjugates), the product will always produce a binomial with a minus sign between the terms.

Example I: $(x - 8)(x + 8)$
 $x^2 + 8x - 8x - 64$
 $x^2 - 64$

PRACTICE

Directions: Simplify.

1 $(-x^3y^2)(-5x^5)$ _____

3 $-5x(x^2 - 9)$ _____

2 $(-4x^2y)(-3y^3)(-x^4y)$ _____

4 $(-8x^2)^3$ _____

5 $(-2xy^2)^2(xy)$ _____

9 $(2x - 6)(2x + 6)$ _____

6 $-6x(x^2 - 3x + 2) + 5(x^2 - 3x + 2)$ _____

10 $(x - 9)(x + 9)$ _____

7 $-5(2x - 3) - 3(x - 2)$ _____

11 $(2x - 5)(3x + 2)$ _____

8 $(x - 7)^2$ _____

12 $(3x - 2)(x + 1)$ _____

6.3 DIVIDING A MONOMIAL OR POLYNOMIAL BY A MONOMIAL



Remember: When dividing and when the bases are the same, subtract the exponents.

$$\frac{x^a}{x^b} = x^{a-b}$$

Apply the above rule when dividing monomials:

- 1) Divide the coefficients.
- 2) Divide like variables by subtracting the exponents.

Example A:

$$\frac{-16x^5y^2}{-4x} \quad \text{First: } -16 \div -4 = 4 \quad \text{Then, } \frac{x^5}{x} = x^{5-1} = x^4$$

Answer is $4x^4y^2$

Example B: Simplify: $\frac{8xy^2}{-16x^2y^2}$

1) Reduce Coefficients: $\frac{8xy^2}{-16x^2y^2} = \frac{1xy^2}{-2x^2y^2}$

2) Reduce Variables: $\frac{1xy^2}{-2x^2y^2} = \frac{1}{-2x}$

Note: Normally, negative exponents are not left in the answer.



Remember: When dividing a polynomial by a monomial, divide each term in the numerator by the monomial in the denominator.

Example C: Simplify: $\frac{-16x^3 - 4x}{-4x}$

Reduce: $\frac{-16x^3 - 4x}{-4x}$

Divide by $-4x$.

Answer: $4x^2 + 1$

Important: The $+ 1$ must be included: $\frac{-4x}{-4x} = 1$

PRACTICE

Directions: Solve each of the following.

1 $\frac{-45abc^2}{-15abc} =$ _____

4 $\frac{28a^3b}{42a^5b} =$ _____

2 $\frac{10a^2b^3}{15ab} =$ _____

5 $\frac{-24a^2 + 12a}{6a} =$ _____

3 $\frac{-14x^3}{7x^5} =$ _____

6 $\frac{7a^2 - 7a}{7a} =$ _____

6.4 FACTORS AND FACTORING AND COMMON MONOMIAL FACTORS

Numbers that divide evenly into a given number are called (1) _____. For example, the factors of 12 are $\{1, 2, 3, 4, 6, 12\}$. A(n) (2) _____ is the biggest factor that goes into all numbers given.

Example A: 24 and 16 have a greatest common factor (GCF) of 8. In this case, both 24 and 16 can be divided by 2, 4, or 8, with 8 being the greatest or largest number. The result is $24 \div 8 = 3$ and $16 \div 8 = 2$.

Example B: x^4 and x^7 have a GCF of x^4 , because $\frac{x^4}{x^4} = 1$ and $\frac{x^7}{x^4} = x^3$.

Note: Always use the exponent with the smaller power.

Example C: Find the greatest common factor (GCF) of $20x^4y$ and $-15x^2y$.

1) Find the GCF of the coefficients.

$$\begin{array}{r} \wedge \\ 2 \ 10 \\ \wedge \\ 2 \ 5 \end{array} \quad \begin{array}{r} 20 \text{ and } -15 \\ \wedge \\ 3 \ 5 \end{array}$$

Note: Use only positive values when finding the GCF.

2) Find the GCF of the variables.

$$\begin{array}{l} x^4y = x \cdot x \cdot x \cdot x \cdot y \\ x^2y = x \cdot x \cdot y \end{array}$$

3) Multiply the two GCFs.

$$5x^2y$$



Note: When factoring a polynomial, always:

- Begin by looking for a GCF.
- Apply the distributive property in reverse. Divide the GCF into all terms.

Example D: Factor: $-8x^2y - 4x$

Solution: The GCF is $-4x$. Divide by the GCF, then put () around the resulting polynomial and place the GCF in front of the ().

1) $\frac{-8x^2y}{-4x}$ and $\frac{-4x}{-4x}$ results in $2xy + 1$

Note: $-4x$ is the GCF

2) $-4x(2xy + 1)$

Multiply the 2 factors.

Example E: Factor: $12x^2 + 4x + 4$

The GCF is 4 because each coefficient can be divided by 4.

$$\frac{12x^2}{4} + \frac{4x}{4} + \frac{4}{4}$$

Factor out the GCF by dividing by 4.

$$4(3x^2 + 1x + 1)$$



Hint: In this section, it was only necessary to find a GCF. However, after reviewing sections 6.5 and 6.6 (to follow), check to see if the expression in the () can be further factored.

PRACTICE

Directions: For questions 1 through 3, find the GCF.

1 (x^3yz) and (xyz) _____

2 $(-18a^2bc^3)$ and $(24abc^2)$ _____

3 $(8xy)(-16x^2y)$ and $(48x)$ _____

Directions: For questions 4 through 8, factor.

4 $-12a^3 + 18a$ _____

7 $3m^2 - 3m + 6$ _____

5 $24b^2c^2 - 20b^3c$ _____

8 $15m^4 - 5m^2 - 25m$ _____

6 $8a^3 - 2a^2 + 4a$ _____

6.5 FACTORING THE DIFFERENCE OF TWO SQUARES



Remember: From Section 6.2, if binomials that are conjugates are multiplied, the product produced is a binomial. Also, it is the difference of two perfect squares which is the result of multiplying a number by its conjugate.

Example A: $(x - 7)(x + 7) = x^2 - 49$

A number such as 49 is called a(n) (1) _____ because it has an integer as its square root. X^2 is a perfect square, because it has an even exponent. Any variable raised to an even exponent is a perfect square.

When a binomial is factored:

- 1) Look for a GCF.
- 2) See if there is a minus sign between two perfect squares.
- 3) If the answer to step 2 is yes, factor the binomial into the sum and difference of the square roots of each of the terms within the binomial.

Example B: Factor: $x^2 - 121$. Since both x^2 and 121 are perfect squares, and since there is a negative (-) sign between the terms, the sum and difference of the square roots of each term can be factored.

$$\sqrt{x^2} = x \quad \sqrt{121} = 11$$

Answer: $(x - 11)(x + 11)$

Example C: Factor: $4x^4 - 9y^2$. Both terms are perfect squares.

Answer: $(2x^2 - 3y)(2x^2 + 3y)$

Example D: Factor: $9x^3 - 36x$. This expression has a GCF which must be factored out first before looking for the difference of perfect squares.

$$\begin{array}{ll} 9x(x^2 - 4) & \text{Take out the GCF.} \\ 9x(x - 2)(x + 2) & \text{Now, factor.} \end{array}$$



Hint: If asked to factor an expression such as the above, the directions might read FACTOR COMPLETELY. This is a hint to look for the GCF first. But remember, not all problems include this hint.

PRACTICE

Directions: Factor completely each of the following. If the binomial cannot be factored, write the word “prime” in the answer space.

1 $x^2 - 169$ _____

6 $49x^2 - y^2$ _____

2 $4x^2 - 9$ _____

7 $m^2 - n^2$ _____

3 $25 - x^2$ _____

8 $4xy^2 - 16x$ _____

4 $1 - a^2b^2$ _____

9 $y^3 - 16y^5$ _____

5 $16m^2 + 9$ _____

10 $25x^2 - 100$ _____

6.6 FACTORING TRINOMIALS AND FACTORING COMPLETELY



Remember: When two binomials are multiplied (Section 6.2), the middle term (also called the linear term) represents the combination of the two original numbers. The last term represents the product of the two numbers. For example,

$$(x + 6)(x - 5)$$

$$6 - 5 \quad 6 \cdot -5$$

$$x^2 + 1x - 30$$



Remember: When a trinomial is factored, do the “reverse” of the above.

- 1) Look for a GCF.
- 2) Put down two sets of () ().
- 3) Fill-in the first term with the square root of the first term in the trinomial.
- 4) Find two numbers that are factors of the constant and combine to give the coefficient of the middle term.

Example A: Factor: $x^2 - 4x - 12$

Factors of 12: 1 and 12, which combine to 11 or 13

3 and 4, which combine to 7 or 1

2 and 6, which combine to 8 or 4



Note: The 6 and 2 factors are chosen because when combined, they equal the middle term and when multiplied, they equal the third term in the trinomial.

Answer: $(x - 6)(x + 2)$

Note: In $x^2 - 4x - 12$, the higher factor (6) has the sign (–) of the coefficient of the middle term (–4).

Example B: Factor $x^2 + 9x + 18$

Factors of 18: 1 and 18, which combine to 17 or 19
3 and 6, which combine to 9 or 3
2 and 9, which combine to 11 or 7



Note: The 6 and 3 factors are chosen because, when combined, they equal the middle term and when multiplied, they equal the third term in the trinomial.

Answer: $(x + 6)(x + 3)$

Example C: Factor: $6x^3 + 18x^2 - 108x$
Factor out the GCF ($6x$) $(6x^3 + 18x^2 - 108x) \div 6x$
Result: $6x(x^2 + 3x - 18)$

Factors of 18: 1 and 18, which combine to 17 or 19
3 and 6, which combine to 9 or 3
2 and 9, which combine to 11 or 7

Factors of $x^2 + 3x - 18$: $(x + 6)(x - 3)$
Add back the GCF: $6x$
Answer: $6x(x + 6)(x - 3)$

PRACTICE

Directions: Factor completely each of the following.

1 $x^2 + 8x + 15$ _____

6 $3a^2 + 24a + 48$ _____

2 $x^2 - 9x - 36$ _____

7 $a^2 - 13a - 48$ _____

3 $x^2 + 10x + 25$ _____

8 $50 + 15t + t^2$ _____

4 $a^3 - 11a^2 - 42a$ _____

9 $-5a^2 - 15a + 50$ _____

5 $a^2 - 2ab - 3b^2$ _____

10 $2ab^2 + 12ab + 16a$ _____

TEST PREP

Directions: Answer all questions in this part. For each multiple choice question, circle the numeral preceding the word or expression that best completes the statement or answers the question. For all open ended questions, clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, and charts, and identify your answer.

- 1 Which of the following is *not* a perfect square?
- (1) $\frac{16}{49}$
 - (2) 40
 - (3) x^4
 - (4) 64
- 2 If one of the factors of $2x^2 - 1x - 6$ is $(2x + 3)$, the other factor must be
- (1) $(x - 2)$
 - (2) $(x + 2)$
 - (3) $(x + 1)$
 - (4) $(x - 1)$
- 3 If the sum of two polynomials is $8x^2 - 3x + 10$ and $4x^2 - 2$ was subtracted from that sum, the result would be
- (1) $2x^2 - 5$
 - (2) $12x^2 - 3x + 8$
 - (3) $4x^2 + 3x + 8$
 - (4) $4x^2 - 3x + 12$
- 4 If the product of two monomials is $-48x^4y^3$ and one of the factors is $-16x^2y$, then the other factor is
- (1) $-3x^2y^2$
 - (2) $3xy$
 - (3) $3x^2y^2$
 - (4) $-3x^2y$
- 5 If $-6x^2 - 4x + 5$ was subtracted from $x^2 - 2x - 3$, the result would be
- (1) $-7x^2 - 2x + 8$
 - (2) $7x^2 + 2x - 8$
 - (3) $7x^2 - 6x + 2$
 - (4) $-7x^2 - 2x + 2$
- 6 If you square $(x - 5)$, then the result is
- (1) $x^2 - 25$
 - (2) $x^2 + 25$
 - (3) $x^2 + 10x + 25$
 - (4) $x^2 - 10x + 25$
- 7 Which of the following is the solution for the following $(-4x^4y^5)^3$?
- (1) $-64x^{12}y^{15}$
 - (2) $-64x^7y^8$
 - (3) $64x^{12}y^{15}$
 - (4) $64x^7y^8$
- 8 Which of the following is not a factor of $-64x^3y$?
- (1) $-8x^3$
 - (2) $-8xy$
 - (3) $16x^3y$
 - (4) $4y^2$

- 9 Twice the sum of $2x$ and 3 can be represented by
- (1) $4x + 3$
 - (2) $4x + 6$
 - (3) $2 + 2x + 3$
 - (4) $2(2x) + 3$
- 10 Which of the following represents the correct factors of $x^2 - x - 42$?
- (1) $(x + 7)(x - 6)$
 - (2) $(x + 6)(x + 7)$
 - (3) $(x - 7)(x - 6)$
 - (4) $(x - 7)(x + 6)$
- 11 Which of the following expressions represents the perimeter of a rectangle whose length is $2x^2 - 10$ and whose width is $x - 8$?
- (1) $2x^2 + x - 18$
 - (2) $4x^2 + 2x - 36$
 - (3) $2x^3 - 16x^2 - 10x + 80$
 - (4) $3x^2 - 18$
- 12 Find the area of a square whose side is represented by $(x+4)$.
- (1) $4x + 16$
 - (2) $x^2 + 8x + 16$
 - (3) $4(x + 4)$
 - (4) $(x + 4)^4$
- 13 Which of the following are the correct factors of $x^2 - 100$?
- (1) $(x + 10)(x + 10)$
 - (2) $(x + 10)(x - 10)$
 - (3) $(x - 10)(x - 10)$
 - (4) $(x \cdot 10)(x \cdot 10)$
- 14 From the sum of $(x^2 - 5x + 9)$ and $(-4x^2 - 2x - 3)$ subtract $(3x^2 - x - 3)$
- 15 Factor completely: $4x^2 - 64$
- 16 Factor completely: $-6x^3 + 18x^2 - 12x$

- 17 If the area of a rectangle is $x^2 - 144$, show how you can calculate the expression that would represent the size of each side.
- 18 If $n + 1$ represents an odd number, show how you would represent the square of the odd number directly below this integer.
- 19 Find the sum of $3x(x - 5)$ and $-5x(x + 2)$.
- 20 Find the perimeter of a rectangle with the length represented by $2x^2 - 5$ and the width represented by $15 - 4x$.