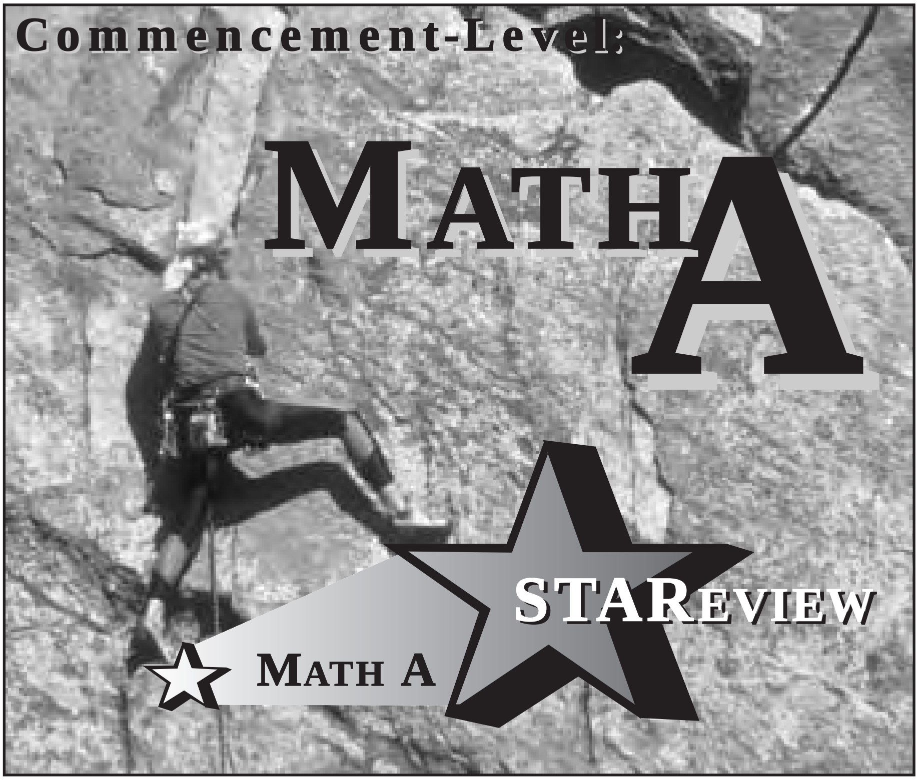

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Dedication
To our family – Dave, Zory, and Maria
Deb, Jeff, Logan, and Lauren
the joy of our lives.

Special Appreciation
Dedicated to our students, with the sincere hope
that they carry throughout their lives the math skills they learn here.

Special Credits
To the many teachers who have contributed their knowledge, skills, and years of experience to the making of our text, we thank you.

To these others, our researchers and readers, our deepest appreciation for their assistance in the preparation of this manuscript.

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phone: 1 800 NN 4 TEXT email: nn4text@nandnpublishing.com

Cat # 960 Soft Cover Edition: ISBN # 0-935487 78 6

2 3 4 5 6 7 8 9 0 BMP 2010 2009 2008 2007 2006 2005
Printed in the United States of America, Book-mart Press, NJ

SAN # 216 - 4221

Table of Contents

Introduction – Solutions to Problems 5
To the Student and Teacher 5

Chapter 1	**Mathematical Reasoning** (Key Idea 1) **7**
Story:	"Who Played the Yankees"? 7
Standard 3.1	*Students use mathematical reasoning to analyze mathematical situations, make conjectures, gather evidence, and construct an argument.* 10
Section 1A	PI [Performance Indicator] – *construct valid arguments* 11
Section 1B	PI – *follow and judge the validity of arguments.* 23
Math Lab	"Logic Circuits" 27
Chapter 2	**Numbers and Numeration** (Key Idea 2) **29**
Story:	"Number Takes Prime Position" 29
Standard 3.2	*Students use number sense and numeration to develop an understanding of the multiple uses of numbers in the real world, the use of numbers to communicate mathematically, and the use of numbers in the development of mathematical ideas.* 31
Section 2A	PI – *understand/use rational and irrational numbers* 31
Section 2B	PI – *recognize the order of real numbers* 38
Section 2C	PI – *apply properties of real numbers to various subsets of numbers* 41
Math Lab	"The Value of Pi" 47
	"TI-83 Calculator" 48
Chapter 3	**Operations** (Key Idea 3) **49**
Story:	"Legend of the Chessboard" 49
Standard 3.3	*Students use mathematical operations and the relationships among them to understand mathematics* 52
Section 3A	PI – *addition, subtraction, multiplication, division, and exponentiation with real numbers and algebraic expressions* 53
Section 3B	PI – *use integral exponents on integers and algebraic expressions* 76
Section 3C	PI – *recognize and identify symmetry and transformations on figures* 79
Section 3D	PI – *use field properties to justify mathematical procedures* 87
Math Lab	"Weight, Gravity, and Distance" 92
Chapter 4	**Modeling and Multiple Representation** (Key Idea 4) **93**
Story:	"You Make the Call" 93
Standard 3.4	*Students use mathematical modeling/multiple representation to provide a means of presenting, interpreting, communicating, and connecting mathematical information and relationships* 95
Section 4A	PI – *represent problem situations symbolically by using algebraic expressions, sequences, tree diagrams, geometric figures, and graphs* 96
Section 4B	PI – *justify the procedures for basic geometric constructions* 118
Section 4C	PI – *use transformations in the coordinate plane* 133
Section 4D	PI – *develop and apply the concept of basic loci to compound loci* 137
Section 4E	PI – *model real-world problems with systems of equations and inequalities* 145
Math Lab	"Lung Capacity" 151

TABLE OF CONTENTS CONTINUED

Chapter 5 **Measurement** (Key Idea 5) **153**

Story: "Home Improvement Project" 153

Standard 3.5 *Students use measurement in both metric and English measure to provide a major link between the abstractions of mathematics and the real world in order to describe and compare objects and data* 155

Section 5A PI – *apply formulas to find measures such as length, area, volume, weight, time, and angle in real-world texts* 155

Section 5B PI – *choose and apply appropriate units and tools in measurement situations* 168

Section 5C PI – *use dimensional analysis techniques* 172

Section 5D PI – *use statistical methods including the measures of central tendency to describe and compare data* 175

Section 5E PI – *use trigonometry as a method to measure indirectly* 185

Section 5F PI – *apply proportions to scale drawings and direct variation* 192

Section 5G PI – *relate absolute value, distance between two points, and the slope of a line to the coordinate plane* 201

Section 5H PI – *explain the role of error in measurement and its consequence on subsequent calculations* 208

Section 5I PI – *use geometric relationships in relevant measurement problems involving geometric concepts* 212

Math Lab "The Carpenter's Square" 214

Chapter 6 **Uncertainty** (Key Idea 6) **215**

Story: "Getting Caught" 215

Standard 3.6 *Students use ideas of uncertainty to illustrate that mathematics involves more than exactness when dealing with everyday situations* 218

Section 6A PI – *judge the reasonableness of results obtained from applications in algebra, geometry, trigonometry, probability, and statistics* ... 218

Section 6B PI – *use experimental and theoretical probability to represent and solve problems involving uncertainty* 223

Section 6C PI – *use concept of random variable in computing probabilities* 228

Section 6D PI – *determine probabilities, using permutations and combinations* 234

Math Labs "Empirical Probability" 241

"Let's Make a Deal" 242

Chapter 7 **Patterns and Functions** (Key Idea 7) **243**

Story: "Homecoming Dance" 247

Standard 3.7 *Students use patterns and functions to develop mathematical power, appreciate the true beauty of mathematics, and construct generalizations that describe patterns simply and efficiently* 245

Section 7A PI –*represent and analyze functions, using verbal descriptions, tables, equations, and graphs* 246

Section 7B PI – *apply linear and quadratic functions in problem solutions* 274

Section 7C PI – *translate among the verbal descriptions, tables, equations, and graphic forms of functions* 279

Section 7D PI – *model real-world situations with the appropriate function* 283

Section 7E PI – *apply axiomatic structure to algebra* 290

Math Lab "The Meaning of Graphs – Are They Possible" 293

Appendices **295**

Symbols, References, and Credits 295

Glossary and Index 295-312

Practice Test 1 (January 2004) 313-326

Practice Test 2 (June 2004) 327-340

Practice Test 3 (August 2004) 341-352

* PI (Performance Indicator)

To the Student & Teacher

Mathematics A STAReview is based on the new standards and assessments for Math A. It is a comprehensive review of the Key Ideas, Major Understandings, Performance Indicators, Process Skills, and Real World situations as set forth in the State of New York Education Department: *Mathematics A – Core Curriculum*.

"Open First"

To begin using this book, you should

- review the Table of Contents (previous 2 pages); this will give you an overview of the major topics reviewed in this book.

- familiarize yourself with the Index & Glossary (in Appendices); this section is an extensive listing of the key mathematical terms needed in order to understand the material; a brief definition or explanation of the term is given together with cross-referenced pages to direct the student to additional material directly related to the term.

Organization

This book is organized conceptually, but the review is linked through the following organizational parts.

- **Key Ideas**. There are seven Key Ideas which correspond to the seven chapters of this book. Key Ideas are used to define the generalized objectives to be reached. Note that each Key Idea flows from a specific Standard in order to help you better understand questions, seek answers, and develop solutions.

- **Standards**. The overall, general goals that apply to all mathematics and indeed most general learning are known as Standards. For example, Standard 3.1 states, *Students use mathematical reasoning to analyze mathematical situations, make conjectures, gather evidence, and construct an argument.*

- **Performance Indicators**. Associated with both the Key Ideas and their corresponding Standards are the Performance Indicators. These tell you specifically what you are expected to know in order to answer correctly the questions on the final, year-end test. In other words, the specific objectives of the testing. All questions included in this *Math A STAReview* fall into one or more of the Performance Indicators.

- **Problems and Solutions**. Each Performance Indicator has specific concepts and mathematical understandings to learn. This is the "meat and potatoes" of *Math A STAReview*. Problems are divided, as they are on the final test, into Parts *1*, *2*, *3*, and *4*.

Meaning of Symbols

Symbols are critical in mathematics. The authors have developed a mini-help system. Stars are used to help you navigate through the more complex major understandings in Mathematics.

Stars ☆ are reminders of important material, identifiers of some special procedures or methods, or perhaps "hints" to help put you on the "right path" or supports for a particular methodology to solve a problem.

Also, stars ☆ indicate two other important things. Some starred material may not be *specifically* referred to in the *Core Curriculum*, but this text is needed for better understanding of major concepts. Also, stars may note special material that further explain Major Understandings, Skills, and Real World Connections. The dash-dot line (–·–·–) in the left margin identifies the extent of this specially identified material.

Finally, Study

Success comes through study. The authors and editors of *Math A STAReview* are teachers. This book has been written to provide you with the best "outside help" possible. But, it can only help you, if you use it consistently, with purpose, and focused study.

We wish you good studying and success on your final test.

Math A – Introduction

Calculators are recommended and required for use on Math A assessments. Scientific calculators are required for the Math A test. Graphing calculators that do not allow for symbolic manipulation are permitted (not required) for the Math A test.

Note: The Math A test may include any given topic listed in the Core Curriculum with any performance indicator.

Chapter 1 [Math A Key Idea 1]

MATHEMATICAL REASONING

for more info: http://www.mlb.com

Who played the Yankees?

There are four baseball games tonight. Three sportswriters predict the winners on the radio.

Slope picks: Blue Jays, Red Sox, Yankees, and Athletics
Powers picks: Devil Rays, Mariners, Yankees, and Blue Jays
Locus picks: Orioles, Red Sox, Mariners, and Blue Jays
No one picks the Tigers

QUESTION: WHO PLAYED THE YANKEES?

Organizing and analyzing information to make logical decisions are important skills for both students and professionals. Doctors, scientists, business managers, and teachers use these skills daily in their work.

Story solution on following page

Solution: Who Played the Yankees?

In solving logic problems, the first step is to gather information and organize it. Every part of the problem is important and may contain important information. The first sentence shows that there are four baseball games and the second sentence indicates that there are three sportswriters making picks.

First, set up a table to help organize the facts.

	Game 1	Game 2	Game 3	Game 4
Sportswriter: Slope				
Sportswriter: Powers				
Sportswriter: Locus				

Next, fill-in the teams that Slope picks (*italic*) for the four games.

	Game 1	Game 2	Game 3	Game 4
Sportswriter: Slope	*Blue Jays*	*Red Sox*	*Yankees*	*Athletics*

The table now shows the names of the four teams that Slope picked to win each game. You now know four of the eight teams playing. Next, fill-in Powers' picks that match Slope's game picks.

	Game 1	Game 2	Game 3	Game 4
Sportswriter: Slope	*Blue Jays*	*Red Sox*	*Yankees*	*Athletics*
Sportswriter: Powers	*Blue Jays*	*?*	*Yankees*	*?*

From the addition of Powers' picks, you know the names of two additional teams: Devil Rays and Mariners. You know that they are playing with the Red Soxs or the Athletics, but you do not know which team played in which game. Next, add Locus' picks that match Slope's and Powers' game picks.

	Game 1	Game 2	Game 3	Game 4
Sportswriter: Slope	*Blue Jays*	*Red Sox*	*Yankees*	*Athletics*
Sportswriter: Powers	*Blue Jays*	*?*	*Yankees*	*?*
Sportswriter: Locus	*Blue Jays*	*Red Sox*	*?*	*?*

The table now shows the common picks of all three sportswriters, and Locus has added the seventh team in his picks, the Orioles. You also know from the last line of the problem that none of the three sportswriters picked the eighth team, the Tigers.

Now apply the logic to fill-in the rest of the table (below). Since the Mariners were referred to in Powers' picks, the Mariners must have participated in either game 2 or 4. Locus' picks indicate that the Mariners must have played in game 3 or 4.

	Game 1	Game 2	Game 3	Game 4
Sportswriter: Slope	*Blue Jays*	*Red Sox*	*Yankees*	*Athletics*
Sportswriter: Powers	*Blue Jays*	Mariners or Devil Rays?	*Yankees*	Mariners or Devil Rays?
Sportswriter: Locus	*Blue Jays*	*Red Sox*	Mariners or Orioles?	Mariners or Orioles?

Combining Powers' and Locus' information shows that the Mariners must have played in game 4, the Orioles played in game 3 and the Devil Rays played in game 2, and the Tigers played the Blue Jays. The line up:

Game 1	Game 2	Game 3	Game 4
Blue Jays vs. Tigers	*Red Sox vs. Devil Rays*	***Yankees vs. Orioles***	Athletics *vs. Mariners*

*****end of solution*****

Principles of Mathematical Logic

Mathematical reasoning solved the question of which teams played each other in the previous example. Now, imagine the world without Game Boys, Play Stations, X boxes, and computers. Did you know that all of these devices are based on the principles of logic?

George Boole, an Englishman born in 1815 developed a system of symbols to deal with complex logic problems. That system was called **Boolean Algebra**, and it is still in use today. Not only does it explain logic problems, but it is indispensable in the design of computer chips and circuits. Without Boolean Algebra, there would be no computer games, cell phone technology, or high performance cars. Even your calculator has a Boolean Chip in it.

As you begin your review of logic, remember that there is no telling how far it will take you. Understanding logic is the first step on the road to understanding both the hardware in the computer and the strategy of the computer game.

Mathematical reasoning is one form of **logic**. In logic, statements are made which can then be evaluated. These statements are called sentences. A **mathematical sentence** is one in which a fact or a complete idea is expressed. Many of these facts can then be judged to be true or false. Questions and phrases are not mathematical sentences because they cannot be judged to be true or false. There are two types of mathematical sentences:

1 A **closed sentence** is a mathematical sentence which can be judged to be true or false. A closed sentence has no variables.

"Hulk is an animated movie character." (is a true closed sentence)

"2 + 5 = 5." (is a false closed sentence)

2 An **open sentence** is a sentence which contains a variable.

"x + 9 = 21" (is an open sentence) – the variable is "*x*."

"It is my favorite course." (is an open sentence) – the variable is "It."

Just as in English class, when a mathematical statement consists of one thought, it is called a simple sentence. When two or more thoughts are connected in one sentence, it is a compound sentence.

"Today is a Saturday." is a simple sentence.

"Today is a Saturday and I slept late." (is a compound sentence)

STANDARD 3
Students use mathematical reasoning to analyze mathematical situations, make conjectures, gather evidence, and construct an argument.

Students:
- construct simple logical arguments.
- follow and judge the validity of logical arguments.
- use symbolic logic in the construction of valid arguments.
- construct proofs based on deductive reasoning.

This is evident, for example, when students:
- prove that an altitude of an isosceles triangle, drawn to the base, is perpendicular to that base.
- determine whether or not a given logical sentence is a tautology (redundancy).
- show that the triangle having vertex coordinates of (0,6), (0,0), and (5,0) is a right triangle.

> **Section 1A – PERFORMANCE INDICATOR 1A**
> Construct valid arguments.
> - Truth value of compound sentences (conjunction, disjunction, conditional, related conditionals such as converse, inverse, and contrapositive, and biconditional).
> - Truth value of simple sentences (closed sentences, open sentences with replacement set and solution set, negations).

Constructing Valid Arguments

Below are tables which summarize the key operations. They will help determine the truth of a closed sentence.

Operation	Key word(s) or process(s)	Results/ hints
Negation	**not**	true becomes false; false becomes true
Conjunction	**and**	only true when both facts are true
Disjunction	**or**	true becomes false; false becomes true

In a compound sentence, the first part of the sentence is referred to as the **hypothesis**. The last part of the sentence is called the **conclusion**. When constructing arguments, these parts can exchange positions in the sentence or be negated. By determining the truth about each fact, the truth of the statement can be determined.

Operation	Key word(s) or process(s)	Results/ hints
Conditional	**if…then**	usually true unless it does not happen
Converse	interchange hypothesis and conclusion	not necessarily true after interchange
Inverse	negates the hypothesis and conclusion	converse and inverse are logically equivalent
Contrapositive	negates and switches places of hypothesis and conclusion	does converse AND Inverse
Biconditional	**if and only if**	true if both true or both false

Negation – Not

A **negation** of a statement is formed by placing the word "not" into the original statement. The negation will always have the opposite truth value of the original statement.

Under negation, what was TRUE will become FALSE, or what was FALSE will become TRUE.

Examples

1 Original Statement: "3 + 4 = 7." (is true)
Negation: "3 + 4 does **not** equal 7." (is false)

2 "A horse is a cow." (is a false statement)
"A horse is **not** a cow." (is a true statement)

"It is not true that a horse is not a cow." (is a false statement [double negative – returns to original value])

Under negation, TRUE becomes FALSE or FALSE becomes TRUE. When *two* negations are used, the truth value of the statement is returned to its original value. This means that two negations will "cancel."

Conjunction – And

A **conjunction** is a compound sentence formed by combining two sentences (or facts) using the word "and." A conjunction is true only when BOTH sentences (or facts) are true.

Examples

1 "Ice is cold and 6 + 6 = 12." (T and T = T)
Since both facts are true, the entire sentence is true.

2 "Twelve inches equals one foot and thirty inches equals one yard." (T and F = F) Since the second fact is false, the entire sentence is false.

3 "All horses moo and all fish bark." (F and F = F)
Since both facts are false, the entire sentence is false.

For a conjunction (and) to be true, BOTH facts must be true.

Venn Diagrams Visualize Conjunctions

The **Venn diagram** is made up of two or more overlapping shapes, usually circles, representing groups of items sharing common properties. The Venn diagram is often used in mathematics to show relationships between sets. (You also may find it useful in other classes. Venn diagrams are useful for examining similarities and differences in characters, stories, and poems.)

Each Venn diagram begins with a rectangle representing the universal set. Then, each group in the problem is represented by a circle. Any components that belong to more than one group are placed in the sections where the shapes overlap.

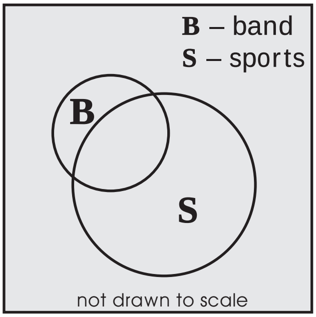
Sharing is illustrated on a Venn diagram where the shapes overlap one another. "Sharing of conditions" makes Venn diagrams useful tools for visualizing problems in both math and other subjects.

Example Problem 1A

In a school of 320 students, 85 students are in the band, 200 students are on sports teams, and 60 students participate in both activities. How many students are involved in either band or sports? Show how you arrived at your answer.

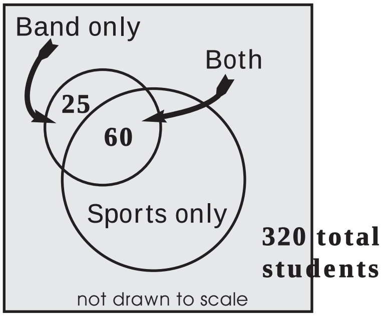
Solution

First, make a Venn diagram to visualize the problem. Identify the rectangle as the entire school. The smaller circle represents the members of the band (B). The larger circle represents the members of sports teams (S). The area where the two intersect represents the students who are members of both teams. **This diagram is not drawn to scale.**



Second, add the data that you were given to the diagram.

Since 60 students from the band also play sports, the students who are in the band only is represented by 85 - 60 = 25 students.



Since 60 students from the sports are also in the band, the students who play sports only are 200 - 60 = 140 students.

The question asked how many students were involved in either band or sports. Add up all the pieces of your Venn diagram.

Students in the band only	25
Students in both	60
Students in sports only	140
Total involved in both	225

Additional Problem

Try a variation on the example problem on the previous page. How many students are involved in neither the band nor sports teams?

Hint: This is represented by the area outside of the circles and inside the rectangle. $320 - 225 = 95$

☆ **Note:** You don't need to use a Venn diagram to solve these problems, but it does help to visualize the situation presented.

Disjunction – Or

A **disjunction** is a compound sentence formed by combining two sentences (or facts) using the word "or." For a disjunction to be true, EITHER or BOTH facts must be true. A disjunction is false when both are false.

Examples

1 "Ice is cold or 6 + 6 = 12." (T or T = T)
Since both facts are true, the entire sentence is true.

2 "Twelve inches equals a foot or thirty inches equals one yard." (T or F = T) Since the first fact is true, the entire sentence is true.

3 "All chickens moo or all fish fly." (F or F = F)
Since both facts are false, the entire sentence is false.

4 "6 + 3 = 12 or red is a color." (F or T = T)
Since the second fact is true, the entire sentence is true.

Conditional – If...Then

A **conditional** is a compound statement formed by combining two sentences (or facts) using the words "if...then." A conditional can also be called an implication.

IF...THEN is only FALSE when the "If" is satisfied, but the "then" doesn't happen. In other words, as long as the "then" is true, the "if" part can be true or false, and the statement is true. "If...then" is true, when both the "if" and the "then" clauses are true.

Example – The Statement

Your teacher tells you that "If you complete your science labs, then you will be able to take the final test."

fact 1 – "you complete your science labs."
fact 2 – "you get to take the final test."

Problem

When is the statement true?

1 "If you complete your science labs (fact 1 true), you get to take the final test." (fact 2 true) then the teacher's statement is true.

2 "If you complete your science labs (fact 1 true), you do not get to take the final test." (fact 2 false), then the teacher did not tell the truth and the statement is false.

3 "If you do not complete your science labs (fact 1 false)..." We cannot judge the truth of the teacher's statement. The teacher did not tell you what would happen if you did NOT complete your science labs. This statement is true since the "then" statement is true or false.

"If you complete your science labs, then you will get to take the final test." This is true in all cases except one: when you complete your science labs and you do NOT get to take the final test.

Biconditional – If and Only If

A **biconditional** is a compound statement formed by combining two conditionals with the word "and." Biconditionals are true when both statements (facts) have the exact same truth value.

A biconditional is read as "[some fact] if and only if [another fact]" and is true when the truth values of both facts are exactly the same – BOTH TRUE or BOTH FALSE.

IF AND ONLY IF is TRUE when both facts are T or both facts are F.

Biconditionals are often used to form definitions.

Definition: A triangle is equilateral if and only if the triangle has three sides of equal length.

The "if and only if" portion of the definition tells you that the statement is true when either fact is the hypothesis. This means that both of the statements below are true.

Example

If a triangle is equilateral, then the triangle has three equal sides (true). If a triangle has three equal sides, then the triangle is equilateral (true).

Other Conditionals – Converse, Inverse and Contrapositive

A "conditional statement" is a statement that can be expressed in "if...then" format. Each of these new conditionals is formed by rearranging an original conditional statement. Review these definitions:

Hypothesis – Hypothesis is the first part of the sentence. It follows the word "If..."

Conclusion – Conclusion is the last part of the sentence. It follows the word "then..."

☆ **Note:** It may be necessary when working with conditional statements to rewrite the sentence so that it is in "If...then" format. For example:

"All downhill skiers like steep trails."
"If you are a downhill skier, then you like steep trails."

Putting this sentence in "If...then" format will make your work easier.

Converse – "Switch"

The **converse** of a conditional statement is formed by interchanging (switching) the hypothesis and conclusion of the original statement. The parts of the sentence change places but the words "if" and "then" do not move.

Example

Conditional – "If the plant lived, then it received sufficient water."

Converse – "If it received sufficient water, then the plant lived."

☆ **Hint:** The logical CONVERSE reminds some students about Converse™ sneakers. The two parts of the sentence "put on their sneakers" and "run" to their new positions. The converse does NOT necessarily have the same truth value as the original conditional statement.

Example

Conditional – "If the plant lived, then it had sufficient water."
This statement is true since water was necessary component for plant life.

Converse – "If the plant had sufficient water, then it lived."
This statement is not always true since many other components (food, temperature, light) are necessary for the plant to live.

☆ **Hint:** The converse has the same truth value as the inverse of the original statement. The CONVERSE and the INVERSE of the original statement are logically equivalent (the same).

INVERSE – "NEGATE"

The **inverse** of a conditional statement is formed by negating the hypothesis and negating the conclusion of the original statement. The word "not" or some other negative word is added to both parts of the sentence.

EXAMPLE

Conditional – "If the plant lived, then it received sufficient water." Considering that the plant needs water, this statement is true.

Inverse – "If the plant did not live, then it did not receive sufficient water." Water is not the only nutrient the plant needs to live. This statement is not always true since many other nutrients and conditions (food, temperature, light) necessary for the plant to live.

☆ **REMINDER:** The inverse has the same truth value as the converse of the original statement. The INVERSE and the CONVERSE of the original statement are logically the same.

CONTRAPOSITIVE – "SWITCH AND NEGATE"

The **contrapositive** of a conditional statement is formed by negating both the hypothesis and the conclusion, and then interchanging the resulting negations. In other words, the contrapositive negates and switches the parts of the sentence. It does BOTH the jobs of the INVERSE and the CONVERSE.

EXAMPLE

Conditional – "If the plant lived, then it received sufficient water."

Contrapositive – "If the plant did not receive sufficient water, then it did not live."

☆ **REMINDER:** The contrapositive is really the combining together of the strategies of two other words: converse and inverse.

The contrapositive always has the SAME truth value as the original conditional statement. If the original statement is TRUE, the contrapositive is TRUE. If the original statement is FALSE, the contrapositive is FALSE. They are said to be logically equivalent (the same).

In the example above, since the conditional statement is true, the contrapositive statement is also true.

Example Question and Solution 1B

"If Mary and Tom are classmates, then they go to the same school."
Which statement below is logically equivalent?

1 If Mary and Tom do not go to the same school, then they are not classmates.
2 If Mary and Tom are not classmates, then they do not go to the same school.
3 If Mary and Tom go to the same school, then they are classmates.
4 If Mary and Tom go to the same school, then they are not classmates.

Solution

First, identify the hypothesis and the conclusion in the original conditional statement:

"If Mary and Tom are classmates, then they go to the same school."
(hypothesis = classmates and conclusion = same school)
Next, negate both the hypothesis and the conclusion (inverse):

"If...not classmates, then...not same school."

Lastly, interchange the positions of the hypothesis and the conclusion (converse).

"If...not same school, then...not classmates."

The correct answer is **1**.

☆ **Remember:** The contrapositive always has the same truth value as the original conditional statement and is said to be logically equivalent.

Part 1 – Practice Question 1A

1 03.6.08 Which statement is logically equivalent to "If it is Saturday, then I am not in school"?
(1) If I am not in school, then it is Saturday.
(2) If it is not Saturday, then I am in school.
(3) If I am in school, then it is not Saturday.
(4) If it is Saturday, then I am in school.

2 03.6.17 What is the inverse of the statement "If Julie works hard, then she succeeds"?
(1) If Julie succeeds, then she works hard.
(2) If Julie does not succeed, then she does not work hard.
(3) If Julie works hard, then she does not succeed.
(4) If Julie does not work hard, then she does not succeed.

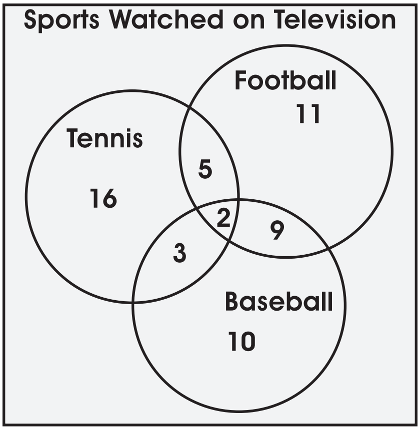
3 03.1.03 What is the inverse of the statement "If Mike did his homework, then he will pass this test"?
(1) If Mike passes this test, then he did his homework.
(2) If Mike does not pass this test, then he did not do his homework.
(3) If Mike does not pass this test, then he only did half his homework.
(4) If Mike did not do his homework, then he will not pass this test.

4 03.1.08 Given the true statement: "If a person is eligible to vote, then that person is a citizen." Which statement must also be true?
(1) Kayla is not a citizen; therefore, she is not eligible to vote.
(2) Juan is a citizen; therefore, he is eligible to vote.
(3) Marie is not eligible to vote; therefore, she is not a citizen.
(4) Morgan has never voted; therefore, he is not a citizen.

5 02.1.14 Frank, George, and Hernando are a plumber, a cabinet maker, and an electrician, though not necessarily in that order. Each can do all work appropriate to his own field, but no work in other fields. Frank was not able to install a new electric line in his home. Hernando was not able to make cabinets. George is also a building contractor who hired one of the other people to do his electrical work. Which statement must be true?
(1) Hernando is an electrician.
(2) George is a cabinet maker.
(3) Frank is a plumber.
(4) Frank is an electrician.

6 01.8.16 Which statement is the converse of "If it is a 300 ZX, then it is a car"?
(1) If it is not a 300 ZX, then it is not a car.
(2) If it is not a car, then it is not a 300 ZX.
(3) If it is a car, then it is a 300 ZX.
(4) If it is a car, then it is not a 300 ZX.

7 02.6.03 The diagram at the right shows the results of a survey asking which sports the members of the Key Club watch on television. Which statement or statements are true? [not drawn to scale]



I The most watched sport is tennis.
II The least watched sport is baseball.
III More Key Club members watch tennis than football.

(1) I, only
(2) II, only
(3) I and II, only
(4) II and III, only

8 01.8.17 In a class of 450 students, 300 are taking a mathematics course and 260 are taking a science course. If 140 of these students are taking both courses, how many students are *not* taking either of these courses?
(1) 30
(2) 40
(3) 110
(4) 140

9 01.6.10 At a school costume party, seven girls wore masks and nine boys did not. If there were 15 boys at the party and 20 students did not wear masks, what was the total number of students at the party?
(1) 30
(2) 33
(3) 35
(4) 42

10 00.8.14 What is the converse of the statement "If it is sunny, I will go swimming"?
(1) If it is not sunny, I will not go swimming.
(2) If I do not go swimming, then it is not sunny.
(3) If I go swimming, it is sunny.
(4) I will go swimming if and only if it is sunny.

11 00.6.06 What is the inverse of the statement "If it is sunny, I will play baseball"?
(1) If I play baseball, then it is sunny.
(2) If it is not sunny, I will not play baseball.
(3) If I do not play baseball, then it is not sunny.
(4) I will play baseball if and only if it is sunny.

12 00.8.06 If $a < b$, $c < d$, and a, b, c, and d are all greater than 0, which expression is always true?
(1) $a - c + b - d = 0$
(2) $a + c > b + d$
(3) $\frac{a}{d} > \frac{b}{c}$
(4) $ac < bd$

PART 2 – PRACTICE QUESTION 1A

1 01.6.21 A school district offers hockey and basketball. The result of a survey of 300 students showed:

120 students play hockey, only
90 students play basketball, only
30 students do not participate in either sport

Of those surveyed, how many students play both hockey and basketball?

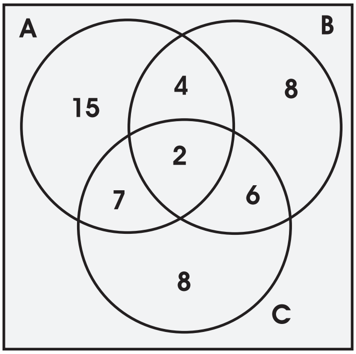
2 Is the inverse of, "if x is positive, then x^2 is positive," always true? Give one example.

3 Write the converse of the following conditional statement, "if a number is divisible by 4, then it is divisible by 2." Then, determine if the converse is true.

PART 3 – PRACTICE QUESTION 1A

1 02.8.26 In a telephone survey of 100 households, 32 households purchased Brand A cereal and 45 purchased Brand B cereal. If 10 households purchased both items, how many of the households surveyed did not purchase either Brand A or Brand B cereal?

2 00.6.26 The accompanying Venn diagram shows the number of students who take various courses. All students in circle A take mathematics. All in circle B take science. All in circle C take technology. [not drawn to scale]



a What number of the students take mathematics or technology?

b What is the total number of students in the sample?

c What is the percentage of students taking mathematics or technology?

3 00.8.26 John, Dan, Karen, and Beth went to a costume ball. They chose to go as Anthony and Cleopatra, and Romeo and Juliet. John got the costumes for Romeo and Cleopatra, but not his own costume. Dan saw the costumes for Juliet and himself. Karen went as Anthony. Beth drove two of her friends, who were dressed as Anthony and Cleopatra, to the ball. What costume did John wear? (Hint: Make a chart to organize the information.)

Section 1B – PERFORMANCE INDICATOR 1B

- Follow and judge the validity of arguments.
- Truth value of compound sentences (conjunction, disjunction, conditional, related conditionals such as converse, inverse, and contrapositive, and biconditional).

Judging Validity of Arguments

In real world situations, logic can help to determine the truth of what is being discussed. Listen closely to statements made and evaluate their validity, logic is not limited to math, but can apply to everything from politics to relationships.

Compound Sentences

A **compound sentence** is formed when two or more thoughts are connected in one sentence. When attempting to determine the truth value of a compound sentence, first determine the truth value of each of the components of the sentence. Next, substitute the truth value for the facts and simplify. Now, look at the three examples listed below.

Example 1

Conjunction (and)

"48 is a multiple of 6 and 26 is an odd number."

Start by determining the truth value of:

48 is a multiple of 6 and 26 is an odd number.
48 is a multiple of 6. (this is true)
26 is an odd number. (this is false)

Now, substitute the truth values for the facts: T and F

☆ **Reminder –** When simplifying, remember that for a conjunction (and) to be true both elements must be true. Therefore, this compound sentence is false. T and F = F

Example 2

Disjunction (or)

"2 + 4 = 6 or 7 – 5 = 12."

Start by determining the truth value of: 2 + 4 = 6 or 7 – 5 = 12.

2 + 4 = 6 (this is true)
7 – 5 = 12. (this is false)

Now, substitute the truth values for the facts: T or F

☆ **Reminder:** When simplifying, remember that for a disjunction (or) to be true, **only one** of the elements needs to be true. Therefore, this disjunction (or) sentence is true. T or F = T

EXAMPLE 3

Conditional (If…then)

"If 3 x 8 = 24, then 4 + 8 = 11"

Start by determining the truth value of: if 3 x 8 = 24, then 4 + 8 = 12

3 x 8 = 24 (this is true)

4 + 8 = 12 (this is true)

Now, substitute the truth values for the facts: if T then T

☆ **REMINDER:** The conditional will be true for the following conditions:

If T, then T = T

If F, then T = T

If F, then F = T

TRUTH VALUE OF OPEN SENTENCES

An **open sentence** is a sentence which contains a variable. A **variable** is simply a word or symbol waiting for a value. The values we put into the variable are called the **domain**, or replacement set (because they "replace" the variable.) The set of values which make the sentence TRUE is called the **solution set**, or **truth set**.

EXAMPLES

Open sentence – "He took the rowboat out on the lake." Note: the variable is "He."

Variable – he

Domain – for example, Logan (the specific person's name).

Solution set – {Logan}

(the specific answer which makes the open sentence true)

Open Sentence	Variable	Domain Replacement Set	Solution Set Truth Set
She has a new job.	She	Lauren, Maria, Deb, and Zori	{Lauren}
$X + 18 = 33$	X	all positive integers	{15}
It is a great strategy game.	It	all board games	{chess}

☆ **NOTE:** Open sentences require that you have additional information to determine whether they are true or false.

PART 1 – PRACTICE QUESTION 1B

1 02.1.20 Which statement is logically equivalent to "If the team has a good pitcher, then the team has a good season"?

(1) If the team does not have a good season, then the team does not have a good pitcher.
(2) If the team does not have a good pitcher, then the team does not have a good season.
(3) If the team has a good season, then the team has a good pitcher.
(4) The team has a good pitcher and the team does not have a good season.

2 01.8.04 Which statement is logically equivalent to "If I did not eat, then I am hungry"?
(1) If I am not hungry, then I did not eat.
(2) If I did not eat, then I am not hungry.
(3) If I am not hungry, then I did eat.
(4) If I am hungry, then I did eat.

3 01.6.12 Which statement is logically equivalent to "If I eat, then I live"?
(1) If I live, then I eat.
(2) If I eat, then I do not live.
(3) I live if and only if I eat.
(4) If I do not live, then I do not eat.

4 01.8.15 If $a + b$ is less than $c + d$, and $d + e$ is less than $a + b$, then e is
(1) less than c
(2) equal to c
(3) less than d
(4) greater than d

PART 2 – PRACTICE QUESTION 1B

1 02.6.21 Given the true statement "John is not handsome" and the false statement "John is handsome or smart." Determine the truth value for the statement "John is smart."

2 Determine the truth value of the following statements:

(1) $12 \div 6 = 2$ and $5 \times 8 = 20$
(2) $2 + 3 - 1 = 4$ or $3 + 2 + 1 = 6$
(3) If $4 \times 12 = 48$, then $(.01)(100) = 1$.

PART 3 – PRACTICE QUESTION 1B

1 Write the contrapositive of: If $x + 2 = 9$, then $x = 7$.

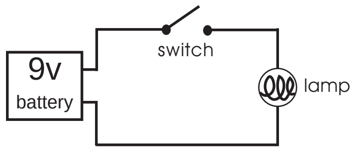
2 In a large high school of 1900 students, 900 students passed chemistry, 800 students passed physics, and 600 students passed chemistry and physics. How many students did not pass either chemistry or physics?

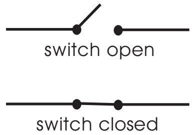
Math Lab – Key Idea 1 – Logical Circuits

Background

A simple electrical circuit is diagramed below:

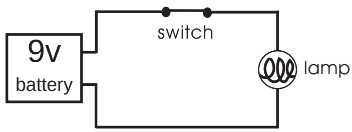
FALSE





When the switch is open, no current flows and the circuit represents a false condition (above).

TRUE



When the switch is closed (at right), current flows and circuit represents a true condition.

Objective

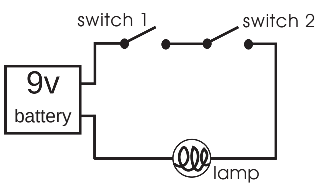
To model logical statements with an electric circuit.

Materials

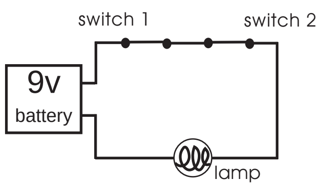
Select from inexpensive, pre-made kits to do this activity or assemble your own components. Suggested: (for each group) 9 Volt battery, battery connectors, Christmas tree lights, connecting wires, switches (or paper clips). [Check with your physics or technology department. It may be possible to borrow some equipment.]

Procedure

1 First set up an "AND" circuit as shown at the right.

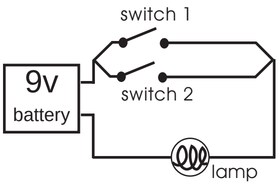


Next, close the two switches as shown at the right.



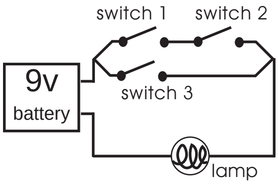
Notice how the path of the wire forms ONE complete loop. BOTH switches must be connected to allow the current to light the bulb. We say the switches are connected in series.

2 Set up an “OR” circuit as shown (Note: Only switch 1 or switch 2 needs to be closed to light the lamp. Trace the two complete paths for the current to follow when each switch is closed independently.



Notice how the path of the wire allows for the forming of TWO complete loops. Connecting either (or both) switches will light the bulb. We say that these switches are connected in parallel.

3 Set up and “AND-OR” circuit as shown. Which switch do you need to close to make the lamp light?



This circuit shows a combination of AND and OR. There are TWO complete loops through which the current can flow. Switch 1 and switch 2 will light the bulb. Switch 3 will light the bulb.

4 Make up several circuits of your own design. Draw the circuits and identify the “and” and “or” components in each.

5 For the statement listed below, set up the circuit which represents the logic. “(When the weather is nice) AND (you are out of school), we will go to the beach.”

6 Make up a logical sentence and arrange the circuit components to illustrate that sentence.

7 Make a circuit model and ask your partner to determine a logical sentence which your circuit could represent. Repeat for each person in the group.

Discussion

In a few well written sentences, explain how electrical circuits can be used to represent a logical mathematical statement.