# Commencement-Level: Math B

## STAReview

### Math B


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**Special Credits**
To the many teachers who have contributed their knowledge, skills,
and years of experience to the making of our text, we thank you.

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* PI (Performance Indicator)

# To the Student & Teacher

***Mathematics B STAReview*** is based on the new standards and assessments for Math B. It is a comprehensive review of the Key Ideas, Major Understandings, Performance Indicators, Process Skills, and Real World situations as set forth in the State of New York Education Department: *Mathematics B – Core Curriculum*.

## "Open First"

To begin using this book, you should

- review the Table of Contents (previous 2 pages); this will give you an overview of the major topics reviewed in this book.
- familiarize yourself with the Index & Glossary (in Appendices); this section is an extensive listing of the key mathematical terms needed in order to understand the material; a brief definition or explanation of the term is given together with cross-referenced pages to direct the student to additional material directly related to the term.

## Organization

This book is organized conceptually, but the review is linked through the following organizational parts.

- **Key Ideas**. There are seven Key Ideas which correspond to the seven chapters of this book. Key Ideas are used to define the generalized objectives to be reached. Note that each Key Idea flows from a specific Standard in order to help you better understand questions, seek answers, and develop solutions.
- **Standards**. The overall, general goals that apply to all mathematics and indeed most general learning are known as Standards. For example, Standard 3.1 states, *Students use mathematical reasoning to analyze mathematical situations, make conjectures, gather evidence, and construct an argument.*
- **Performance Indicators**. Associated with both the Key Ideas and their corresponding Standards are the Performance Indicators. These tell you specifically what you are expected to know in order to answer correctly the questions on the final, year-end test. In other words, the specific objectives of the testing. All questions included in this *Math B STAReview* fall into one or more of the Performance Indicators.
- **Problems and Solutions**. Each Performance Indicator has specific concepts and mathematical understandings to learn. This is the "meat and potatoes" of *Math B STAReview*. Problems are divided, as they are on the final test, into Parts *1*, *2*, *3*, and *4*.

## Meaning of Symbols

Symbols are critical in mathematics. The authors have developed a mini-help system. Stars are used to help you navigate through the more complex major understandings in Mathematics.

Stars are reminders of important material, identifiers of some special procedures or methods, or perhaps "hints" to help put you on the "right path" or supports for a particular methodology to solve a problem.

Also, stars indicate two other important things. Some starred material may not be *specifically* referred to in the *Core Curriculum*, but this text is needed for better understanding of major concepts. Also, stars may note special material that further explain Major Understandings, Skills, and Real World Connections.

## Finally, Study

Success comes through study. The authors and editors of *Math B STAReview* are teachers. This book has been written to provide you with the best "outside help" possible. But, it can only help you, if you use it consistently, with purpose, and focused study.

We wish you good studying and success on your final test.

# Math B – Introduction

**Calculators** are recommended and required for use on Math B assessments. Scientific calculators are required for the Math B test. Graphing calculators that do not allow for symbolic manipulation are permitted (not required) for the Math B test.

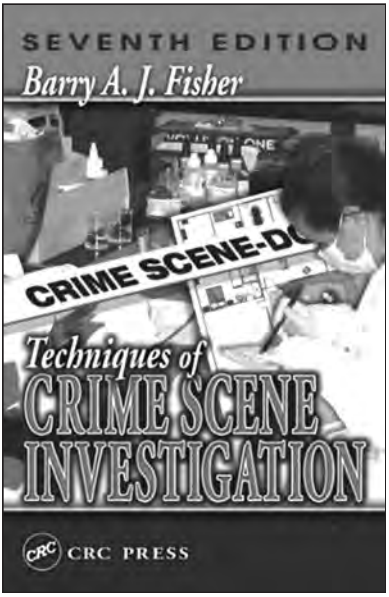
**Note**: The Math B test may include any given topic listed in the Core Curriculum with any performance indicator.

Chapter 1 [Math B Key Idea 1]

# MATHEMATICAL REASONING

## Examination of the Crime Scene

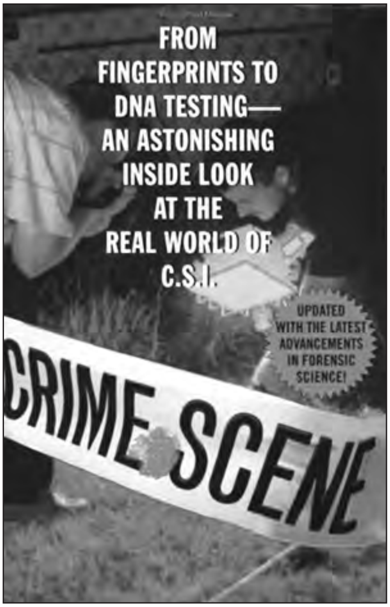
The following is an excerpt from the Louisiana State Police Crime Laboratory




Before the investigators begin examining the scene of the crime, they should gather as much information as possible about the scene. Once again, a slow and methodical approach is recommended. Once all of the information is gathered, a mental plan is formulated as to how the crime scene will be analyzed. Copious notes and relevant times should be kept on every aspect of the crime scene investigation. The examination of the scene will usually begin with a walk through of the area along the "trail" of the crime. The trail is that area which all apparent actions associated with the crime took place. The trail is usually marked by the presence of physical evidence. This may include the point of entry, the location of the crime, areas where a suspect may have cleaned up, and the point of exit.

The purpose of the walk through is to note the location of potential evidence and to mentally outline how the scene will be examined. The walk through begins as close to the point of entry as possible. The first place the investigators should examine is the ground on which they are about to tread. If any evidence is observed, then a marker should be placed at the location as a warning to others not to step on the item of interest.

*Source:* http://leap.ulm.edu/LaGin/pscorre/lspcl.htm




As the walk through progresses, the investigators should make sure their hands are occupied by either carrying notebooks, flashlights, pens, etc. or by keeping them in their pockets. This is to prevent depositing of unwanted fingerprints at the scene. As a final note on the walk through, the investigators should examine whatever is over their heads (ceiling, tree branches, etc.). These areas may yield such valuable evidence as blood spatters and bullet holes. Once the walk through is completed, the scene should be documented with videotape, photographs, and/or sketches.

You have just been requested to help solve a crime mystery. To solve the crime, you take the known facts and, step by step, determine who committed the crime. You provide supporting evidence for each statement you make. Amazingly, this is the same process you use in geometry to solve a proof. The following five steps will take you through the whole process.

**1 Determine the statement of the theorem.**

The statement is what needs to be proven. In the case of the crime, it is who committed it. In the case of geometry, it usually refers to a mathematical statement concerning relationships.

**2 State the given.**

The GIVEN is the hypothesis and contains all the facts that are provided. At the crime scene, it is the evidence and witnesses statements. The given is the information you have been provided with to solve this proof. In geometry, the given is generally written in an area above the proof.

**3 Create a drawing that represents the given.**

Just like the detectives, you need a good picture of the problem. Look at all the information that is provided and draw a figure. Make it large enough that it allows you to put in all the detailed information. Be sure to label all the points with the appropriate letters. If lines are parallel, or if angles are congruent, include those markings, also.

**4 State what you are going to prove.**

The PROVE is where you state what you are trying to show to be true. Like the given, the prove statement is also written in an area above the proof. If it references parts in your figure, so be sure to include the info from the prove statement in your figure. The last line in the statements column of each proof matches the prove statement.

**5 Prove it.**

The proof is a series of logically deduced statements — a step-by-step list that takes you from the given; through definitions, postulates, and previously proven theorems; to the prove statement.

Additional points to remember:

- The given is not necessarily the first information you put into a proof. The given info goes wherever it makes the most sense. The timing of the prosecution's case may not present all of the evidence at a preliminary hearing. Likewise, it may also make sense to put it into the proof at a later step.

- Think of proofs as steps to solving the crime. The object of the proof is to have all the statements in your chain linked so that one fact leads to another until you reach the prove statement. However, before you start the formal proof, you should look over the given and the prove parts, and develop a plan on how to prove your case. Once you decide on a strategy, you can proceed statement by statement, carefully documenting every move in successively numbered steps. All statements you make must refer back to your figure and finally end with the prove statement. The last line under the Statements column should be exactly what you wanted to prove.

**STANDARD 3**

Students use mathematical reasoning to analyze mathematical situations, make conjectures, gather evidence, and construct an argument.

Students:

- construct indirect proofs or proofs using mathematical induction
- investigate and compare the axiomatic structures of various geometries

This is evident, for example, when students:

- prove indirectly that: if $n^2$ is even, n is even.
- prove using mathematical induction that: $1 + 3 + 5 + \ldots + (2n - 1) = n^2$.
- explain the axiomatic differences between plane and spherical geometries.

**Section 1A – PERFORMANCE INDICATOR 1A**

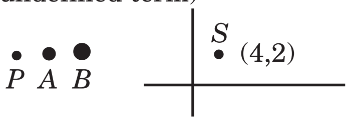
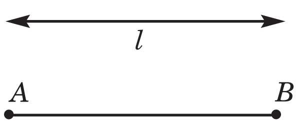
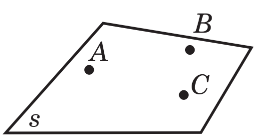
Construct proofs based on deductive reasoning.

- Euclidean and analytic direct proofs.

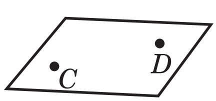
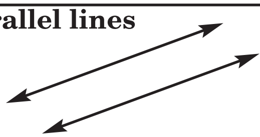
# Getting Ready For Proofs

## Terms And Concepts Of Geometry

An important aspect of geometry is the proof. Definitions, concepts and vocabulary provide the basis for understanding and writing proofs. Some terms are undefined. These terms include the point, the line, and the plane.

| | |
|---|---|
| **Point**<br>(undefined term)<br><br> | A **point** has no length, width, or thickness. It is identified by a capital letter. In a coordinate plane, a point is identified by its coordinates $(x,y)$. |
| **Line**<br>(undefined term)<br><br> | A **line** has no thickness, but it extends forever in both directions in one dimension. A line is usually defined by a lower case letter below the line segment which has arrowheads in both directions or by two points on the line. |
| **Plane**<br>(undefined term)<br><br> | A **plane** has no thickness and extends forever in all directions. A plane is identified by a small letter or three non collinear (lying on the same line) points. |

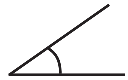
The following definitions are basic concepts in geometry but are seldom used as reasons in formal proofs.

| | |
|---|---|
| **Collinear points**<br><br> | Points that lie in the same line. |
| **Coplanar points**<br><br> | Points that lie on the same plane. |
| **Parallel lines**<br> | Two coplanar lines, that do *not* intersect. |

## VOCABULARY FOR PROOFS

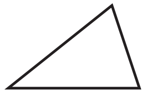
**acute angle**

An acute angle is an angle whose measurement is greater than 0° and less than 90º.



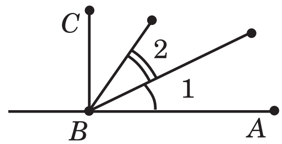
**acute triangle**

An acute triangle is a triangle that has three acute angles.



**adjacent angles**

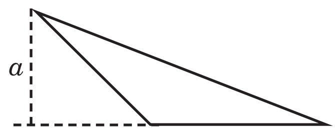
$\angle 1$ and $\angle 2$ are adjacent




Adjacent angles share a common vertex, a common side, but no common interior points.

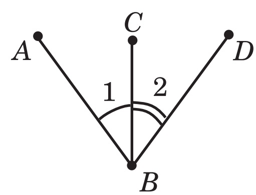
**altitude for an obtuse triangle**

The altitude ($a$) of a triangle is a line segment extending from any vertex of a triangle perpendicular to the line containing the opposite side.




**angle bisector**

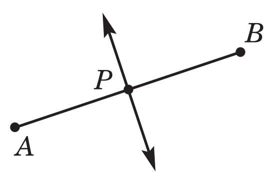
*BC* bisects angle *ABD* and $\angle 1 \cong \angle 2$




An angle bisector is a ray whose endpoint is the vertex of the angle and which divides the angle into two congruent angles.

**bisector**

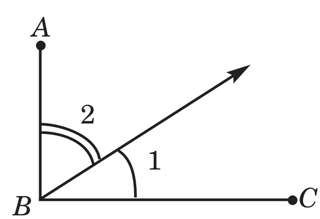
*P* is the midpoint of line *AB*




A bisector of a line segment is any line segment, ray, or plane that intersects the segment at its midpoint.

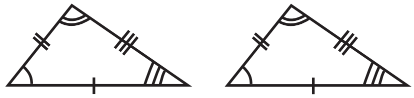
**complementary angles**

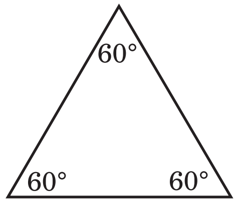
<1 and <2 are complementary




Complementary angles are two angles whose sum is 90º. Complementary angles do not have to be adjacent angles.

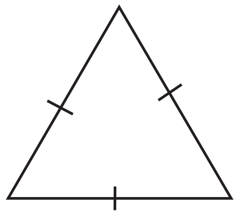
**congruent objects**
Two objects that have the same size and shape are congruent objects.



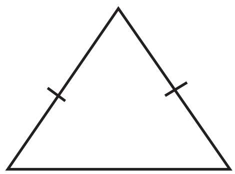



**equiangular triangle**
An equiangular triangle is a triangle that has three congruent angles.

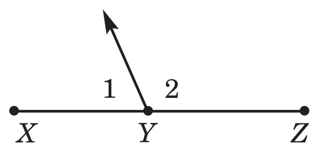
**equilateral triangle**
An equilateral triangle is a triangle with three congruent sides. Equilateral triangles are also equiangular.



**isosceles triangle**
An isosceles triangle is a triangle with two congruent sides.



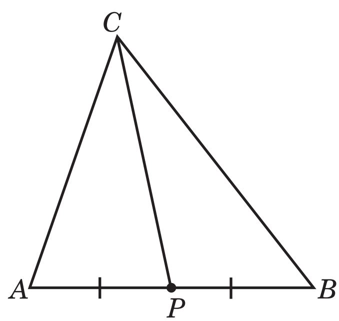
**linear pair**
A linear pair of angles are adjacent angles whose non-common sides form a straight line. The sum of the measurements of the angles in a linear pair is 180º.



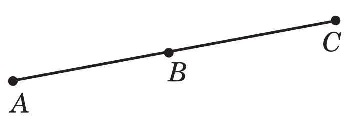

median

**median**
The median of a triangle is a line segment extending from any vertex of a triangle to the midpoint of the opposite side.

$\overline{CP}$ is a median of $\Delta ABC$ and $P$ is the midpoint of $\overline{AB}$
Note: $\Delta ABC$ *is* not isosceles.

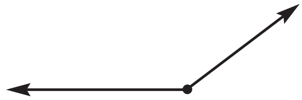



**midpoint**
Midpoint of a line segment is the point on that line segment that divides the segment into two congruent segments.

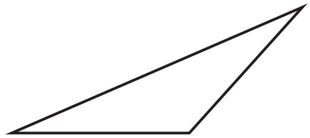



$B$ is the midpoint of line $\overline{AC}$ and $\overline{AB} \cong \overline{BC}$

**obtuse angle**
An obtuse angle is an angle whose measurement is greater than 90º and less than 180º.

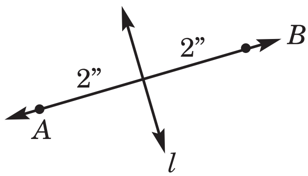


**obtuse triangle**
An obtuse triangle is a triangle which contains one obtuse angle.



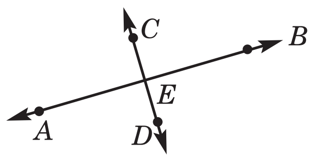
**perpendicular bisector**
The perpendicular bisector of a segment is a line (or subset of a line) that bisects the segment and is perpendicular to the segment.

$l \perp \overleftrightarrow{AB}$ and $l$ bisects $\overleftrightarrow{AB}$

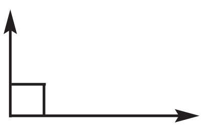



**perpendicular lines**
Perpendicular lines are two lines that intersect to form right angles.

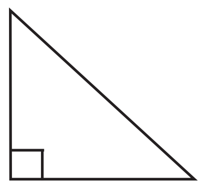
$\overleftrightarrow{AB} \perp \overleftrightarrow{CD}$ and $\angle AEC$, $\angle CEB$, $\angle BED$ and $\angle DEA$ are right angles



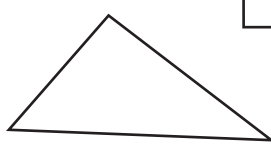

**right angle**
A right angle is an angle whose measurement is 90º.



**right triangle**
A right triangle is a triangle containing one right angle.



**scalene triangle**
A scalene triangle is a triangle with no congruent sides.

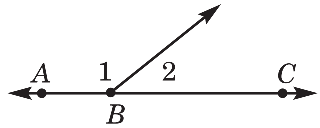


**segment**
A segment is named by its endpoints. $X$ and Z are endpoints of $\overline{XZ}$. $\overline{XZ}$ and $\overline{ZX}$ are the same line segment.

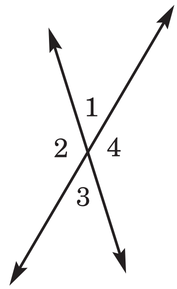

**space**
The set of all points.

**straight angle**
A straight angle is an angle whose measurement is 180º.

**supplementary angles**
$\angle 1$ and $\angle 2$ are supplementary
Supplementary angles are two angles the sum of whose measurements are 180º. (Supplementary angles need not be adjacent.)




**vertical angles**
$\angle 1$ and $\angle 3$ are vertical angles
$\angle 2$ and $\angle 4$ are vertical angles
Vertical angles are two nonadjacent angles formed by two intersecting lines.




## PROPERTIES, POSTULATES, AND THEOREMS FOR PROOFS

This is a listing of the more popular theorems, postulates, and properties needed when working with Euclidean and analytic direct proofs. Your teacher may add to this list or refer to the theorem by a slightly different name. Some properties used in geometry apply to both equality and congruence. The properties of real numbers help us write equivalent expressions. These properties will sometimes help us figure out the value of an unknown from information about it.

| **Property** | **Equality** (numbers, variables, lengths, angle measures) | **Congruence** (segments, angles, polygons) |
|---|---|---|
| Reflexive | A quantity is equal to itself.<br>$DE = DE$<br>$m\angle 2 = m\angle 2$ | A quantity is congruent to itself.<br>$\overline{DE} \cong \overline{DE}$<br>$\angle 2 \cong \angle 2$ |
| Symmetric | If $DE = AB$, then $AB = DE$<br>If $m\angle 1 = m\angle 2$, then $m\angle 2 = m\angle 1$ | If $\overline{DE} \cong \overline{AB}$, then $\overline{AB} \cong \overline{DE}$<br>If $\angle 1 \cong \angle 2$, then $\angle 2 \cong \angle 1$ |
| Transitive | If $AB = CD$ and $CD = EF$ then $AB = EF$<br>If $m\angle 1 = m\angle 2$ and $m\angle 2 = m\angle 3$ then $m\angle 1 = m\angle 3$ | If $\overline{AB} \cong \overline{CD}$ and $\overline{CD} \cong \overline{EF}$ then $\overline{AB} \cong \overline{EF}$<br>If $m\angle 1 \cong m\angle 2$ and $m\angle 2 \cong m\angle 3$ then $m\angle 1 \cong m\angle 3$ |

## POSTULATES

| | |
|---|---|
| Addition Postulate | If equal quantities are added to equal quantities, the sums are equal. |
| Subtraction Postulate | If equal quantities are subtracted from equal quantities, the differences are equal. |
| Multiplication Postulate | If equal quantities are multiplied by equal quantities, the products are equal. Doubles of equal quantities are equal. |
| Division Postulate | If equal quantities are divided by equal nonzero quantities, the quotients are equal. Halves of equal quantities are equal. |
| Substitution Postulate | A quantity may be substituted for its equal in any expression. |
| Parallel Postulate | If there is a line and a point not on the line, then there exists one line through the point parallel to the given line. |
| Corresponding Angles Postulate | If two parallel lines are cut by a transversal, then the pairs of corresponding angles are congruent. |
| Corresponding Angles Converse Postulate | If two lines are cut by a transversal and the corresponding angles are congruent, the lines are parallel. |
| Side-Side-Side (SSS) Congruence Postulate | If three sides of one triangle are congruent to three sides of another triangle, then the triangles are congruent |
| Side-Angle-Side (SAS) Congruence Postulate | If two sides and the included angle of one triangle are congruent to the corresponding parts of another triangle, the triangles are congruent. |
| Angle-Side-Angle (ASA) Congruence Postulate | If two angles and the included side of one triangle are congruent to the corresponding parts of another triangle, the triangles are congruent |
| Angle-Angle (AA) Similarity Postulate | If two angles of one triangle are congruent to two angles of another triangle, the triangles are similar. |

## Theorems

| | |
|---|---|
| Right Angles | All right angles are congruent. |
| Congruent Adjacent Angles | If two lines are perpendicular, then they form congruent adjacent angles. |
| Congruent Supplements | If two angles are supplementary to the same angle or to congruent angles, then the two angles are congruent. |
| Congruent Complements | If two angles are complementary to the same angle or to congruent angles then the two angles are congruent. |
| Vertical Angles | Vertical angles are congruent. |
| Alternate Interior Angles | If two parallel lines are cut by a transversal, then the alternate interior angles are congruent. |
| Alternate Exterior Angles | If two parallel lines are cut by a transversal, then the alternate exterior angles are congruent. |
| Interiors on Same Side | If two parallel lines are cut by a transversal, the interior angles on the same side of the transversal are supplementary. |
| Parallel Lines | Two lines parallel to a third line are parallel to each other. |

## Angle Converse Theorems

| | |
|---|---|
| Alternate Interior Angles Converse | In a plane, if two lines are cut by a transversal and the alternate interior angles are congruent, the lines are parallel. |
| Alternate Exterior Angles Converse | In a plane, if two lines are cut by a transversal and the alternate exterior angles are congruent, the lines are parallel. |
| Congruent Adjacent Angles Converse | If two intersecting lines form congruent adjacent angles, then the lines are perpendicular. |
| Interiors on Same Side Converse | In a plane, if two lines are cut by a transversal and the interior angles on the same side of the transversal are supplementary, the lines are parallel. |

## Theorems For Parallelograms

| | |
|---|---|
| Opposite sides | If a quadrilateral is a parallelogram, the opposite sides are congruent. |
| Opposite angles | If a quadrilateral is a parallelogram, the opposite angles are congruent. If a quadrilateral is a parallelogram, any two consecutive angles are supplementary. |
| Diagonals | If a quadrilateral is a parallelogram, the diagonals bisect each other. If a quadrilateral is a parallelogram, a diagonal divides it into two triangles. |

## PARALLELOGRAM CONVERSES

| | |
|---|---|
| Sides | If both pairs of opposite sides of a quadrilateral are congruent, the quadrilateral is a parallelogram. |
| Angles | If both pairs of opposite angles of a quadrilateral are congruent, the quadrilateral is a parallelogram. If all the points of the consecutive angles of a quadrilateral are supplementary, the quadrilateral is a parallelogram. |
| Diagonals | If the diagonals of a quadrilateral bisect each other, the quadrilateral is a parallelogram. |

### WAYS TO PROVE LINES PARALLEL

- Show the a pair of corresponding angles are congruent.
- Show that a pair of alternate interior angles are congruent.
- Show that a pair of same side interior angles are supplementary.
- In a plane, show that both lines are perpendicular to a third line.
- Show both lines parallel to a third line.

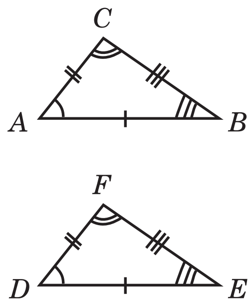
## TRIANGLES AND THEOREMS

| | |
|---|---|
| Triangle Sum | The sum of the interior angles of a triangle is 180°. |
| Exterior Angle | The measurement of an exterior angle of a triangle is equal to the sum of the measurements of the two non-adjacent interior angles. |
| Angle-Angle-Side (AAS) Congruence | If two angles and the non-included side of one triangle are congruent to the corresponding parts of another triangle, the triangles are congruent. |
| Base Angle Theorem (Isosceles Triangle) | If two sides of a triangle are congruent, the angles opposite these sides are congruent. |
| Base Angle Converse (Isosceles Triangle) | If two angles of a triangle are congruent, the sides opposite these angles are congruent. |
| Mid-segment Theorem | The segment connecting the midpoints of two sides of a triangle is parallel to the third side and is half as long. |
| Side Proportionality | If two triangles are similar, the corresponding sides are in proportion. |

## THEOREMS FOR CONGRUENT TRIANGLE PROOFS

**Definition**: Two triangles are congruent if all pairs of corresponding sides are congruent, and all pairs of corresponding angles are congruent.

Proving that two triangles are congruent to each other is a common occurrence in formal proofs. Sometimes this task is the end result of a problem. In other instances, the two congruent triangles could be used to further prove corresponding pairs of angles or line segments to be congruent.




In the diagram at the right, $ABC \cong DEF$.

Since these triangles are congruent, all corresponding sides are congruent to each other, and all corresponding angles are congruent to each other. In the above illustration, the six corresponding parts are:

$\angle A = \angle D$ $\quad$ $\overline{AB} \cong \overline{DE}$

$\angle B = \angle E$ $\quad$ $\overline{AC} \cong \overline{DF}$

$\angle C = \angle F$ $\quad$ $\overline{CB} \cong \overline{FE}$

Note: *C.P.C.T.C. is the abbreviation for Corresponding Parts of Congruent Triangles are Congruent. It is often used as a reason in formal proofs.*

To prove congruency, it is only necessary to show three sets of corresponding parts of the triangles are congruent. There are five methods to choose from to prove that two triangles are congruent to one another.

## METHODS OF PROVING TRIANGLES TO BE CONGRUENT

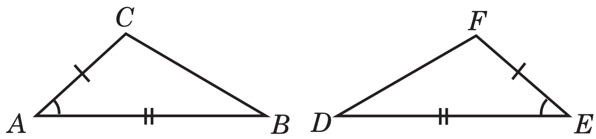
| Method Symbol | Description |
|---|---|
| SSS | If three sides of one triangle are congruent to three sides of another triangle, the triangles are congruent. |
| SAS | If two sides and the included angle of one triangle are congruent to the corresponding parts of another triangle, the triangles are congruent. |
| ASA | If two angles and the included side of one triangle are congruent to the corresponding parts of another triangle, the triangles are congruent. |
| AAS | If two angles and the non-included side of one triangle are congruent to the corresponding parts of another triangle, the triangles are congruent. |
| HL | If the hypotenuse and leg of one right triangle are congruent to the corresponding parts of another right triangle, the right triangles are congruent. |

## METHODS OF PROVING TRIANGLES TO BE CONGRUENT

**EXAMPLE** Proving that triangles are congruent

**GIVEN**

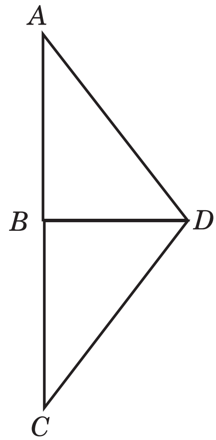
$\overline{AC} \cong \overline{EF}$
$\overline{AB} \cong \overline{ED}$
$\angle A \cong \angle E$




**THEREFORE** $\Delta ABC \cong EDF$ by SAS – side-angle-side.

For each of the following problems, identify which method is used to prove congruency in triangles.

**EXAMPLE 1**




**GIVEN** $\overline{AD} \cong \overline{CD}$

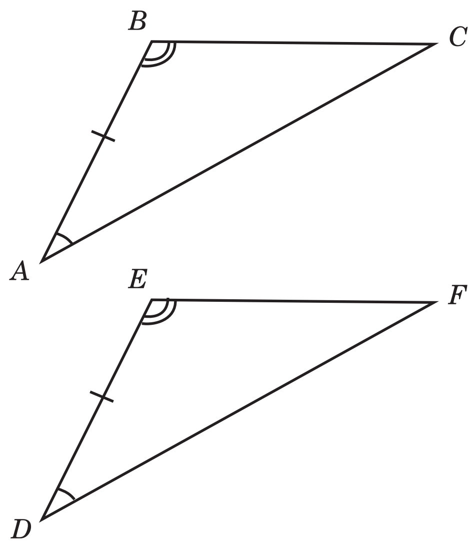
$\overline{AB} \cong \overline{BC}$

$\angle ABD = \angle CBD = 90^{\circ}$

**THEREFORE** $\overline{BD} \cong \overline{BD}$ by reflexive property
$\Delta ABD \cong \Delta CBD$

**REASON** __________________________

**EXAMPLE 2**




**GIVEN** $AB \cong DE$
$\angle A \cong \angle D$
$\angle B \cong \angle E$

**THEREFORE** $\Delta ABC \cong \Delta DEF$

**REASON** __________________________

## THE FORMAL PROOF

When writing a proof, be sure that your argument is clearly developed and that each step is supported by a property, theorem, postulate or definition. Remember that you must always write a proof as if the reader knows nothing about geometry.

1 Draw a good figure that pictures the data or the theorem.
2 State the *givens* in terms of the lettered figure.
3 State the *prove* in terms of the lettered figure.
4 Present the proof, which is a series of logical arguments in proper chronological order, in relation to earlier steps.
5 Each step should consist of a *statement* and its *reason*. A reason may be the *given*, a *definition*, a *postulate,* or a *previously proven theorem*.

## THE TWO-COLUMN (T) PROOF

- The proof itself looks like a big letter "T." The T makes two columns. Put a "statements" label over the left column and a "reasons" label over the right column. Two columns are presented where the first column contains a numbered chronological list of steps ("statements") leading to the desired conclusion.

- Proceed statement by statement, carefully documenting every move in successively numbered steps. All statements that you make must refer back to your figure and finally end with the prove statement. The last line under the Statements column should be exactly what you wanted to prove.

| Statements | Reasons |
|---|---|
| 1 | 1 |
| 2 | 2 |
| 3 | 3 |
| 4 | 4 |

## GETTING STARTED

Before you can write a proof, you need to have a plan for the proof. Sometimes you will see how to do the proof immediately. At other times, you may need to try several times before you find a plan which works. Sometimes you will look forwards and sometimes you will need to work backwards by examining the conclusion to determine how to get there.

Below are two methods to help you find the plan:

**METHOD 1**

- Gather as much information as you can. Sometimes what you see will show you a plan.
- Re-read the "given." What did it tell you?
- Look at the diagram. What other information can you conclude?

**METHOD 2**

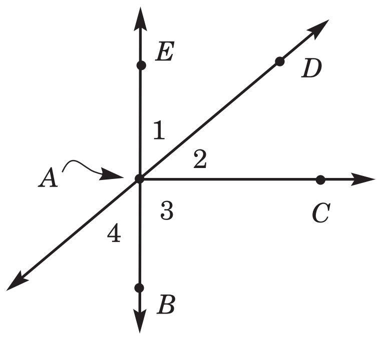
- Work backwards. Look at the conclusion first.
- Consider: This conclusion would be true if __________ . And __________ would be true if __________.
- Continue working backwards until you have a plan.

## EXAMPLE: METHOD 1

**GIVEN**

$\overrightarrow{AD}$ bisects $\angle EAC$

Prove $\angle 2 \cong \angle 4$




**PLAN**

From the given, you can conclude $\angle 1 \cong \angle 2$.

From the diagram you can see that $\angle 1$ and $\angle 4$ are vertical angles so $\angle 1 \cong \angle 4$.

Since $\angle 1 \cong \angle 2$ and $\angle 1 \cong \angle 4$, you can conclude $\angle 2 \cong \angle 4$.

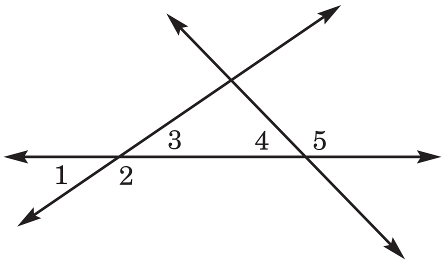
Now write a formal proof.

| STATEMENTS | REASONS |
|---|---|
| 1 $\overrightarrow{AD}$ bisects $\angle EAC$ | 1 Given |
| 2 $\angle 1 \cong \angle 2$ | 2 Definition of angle bisector |
| 3 $\angle 1 \cong \angle 4$ | 3 Vertical Angles Theorem |
| 4 $\angle 2 \cong \angle 4$ | 4 Substitution Postulate |

## Example: Method 2

**Given** $m\angle 3 = m\angle 4$

**Prove** $\angle 5$ is supplementary to $\angle 1$




**Plan**

$\angle 5$ is supplementary to $\angle 1$ if $m\angle 1 + m\angle 5 = 180$.

This is true if $m\angle 4 + m\angle 5 = 180$.

From the diagram $\angle 1 = \angle 3$ and $m\angle 3 = m\angle 4$ so $m\angle 4 = m\angle 1$

| Statements | | Reasons | |
|---|---|---|---|
| 1 | $m\angle 3 = m\angle 4$ | 1 | Given |
| 2 | $m\angle 3 = m\angle 1$ | 2 | Vertical Angles Theorem |
| 3 | $m\angle 5 + m\angle 4 = 180$ | 3 | Definition of Supplement<br>$\angle 4$ and $\angle 5$ form a linear pair |
| 4 | $m\angle 5 + m\angle 3 = 180$ | 4 | Substitution |
| 5 | $m\angle 5 + m\angle 1 = 180$ | 5 | Substitution |

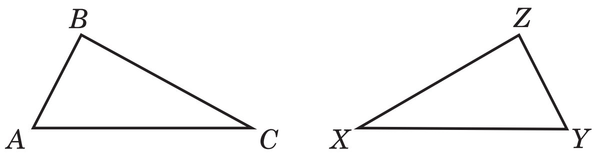
The sum of the angles measured of a linear pair totals 180°.

## Practice With Formal Congruent Triangle Proofs

**Directions** When attempting to prove triangles congruent, it is important to satisfy all of the conditions of the congruent triangle method you are using. In each problem below, examine the diagram and the given information. You may wish to draw the diagrams on separate paper so that you can mark off the information.

- Determine the method needed to prove the triangles congruent. (ASA, SAS, AAS, SSS, or HL for right triangles only)
- Each of the three components needed to support the chosen method appear to the left of their corresponding Statement.
- Decide what Reasons can be used to support your decisions.

## EXAMPLE



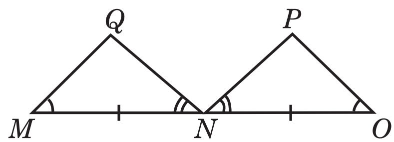

**GIVEN** $\overline{AB} \cong \overline{YZ}$
$\angle B \cong \angle Z$
$\overline{BC} \cong \overline{XZ}$

**PROVE** $\Delta ABC \cong \Delta YZX$

| STATEMENTS | REASONS |
|---|---|
| 1 $\overline{AB} \cong \overline{YZ}$ | 1 Given |
| 2 $\angle B \cong m\angle Y$ | 2 Given |
| 3 $\overline{BC} \cong \overline{XZ}$ | 3 Given |
| 4 $\Delta ABC \cong \Delta YZX$ | 4 SAS ≅ SAS |

## TRY IT: SAMPLE 1

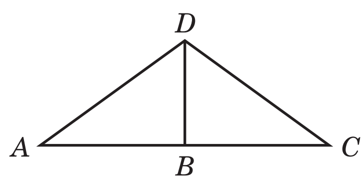
**GIVEN** $\angle M \cong \angle O$
$\angle QNM \cong \angle PNO$
$N$ is the midpoint of $\overline{MO}$




**PROVE** $\Delta MNQ \cong \Delta ONP$

## TRY IT: SAMPLE 2

**GIVEN** $\angle ABD$, $\angle CBD$ are right triangles
$\overline{AD} \cong \overline{DC}$




**PROVE** $\Delta ABD \cong \Delta CBD$

| STATEMENTS | REASONS |
|---|---|
| 1 $\Delta ABD$ and $\Delta CBD$ are right triangles | 1 Given |
| 2 $\overline{AD} \cong \overline{CD}$ | 2 Given |
| 3 $\overline{BD} \cong \overline{BD}$ | 3 Reflexive property of congruence |
| 4 $\Delta ABD \cong \Delta CBD$ | 4 HL |

## Proofs In Coordinate Geometry

Analytic geometry was born in the seventeenth century, when the French mathematician René Descartes applied algebraic principles to geometric situations. This process often involves placing geometric figures in a coordinate plane. It is also referred to as coordinate geometry.

Coordinate geometry proofs employ the use of formulas such as the Distance Formula, the Slope Formula, and/or the Midpoint Formula as well as postulates, theorems, and definitions.

**Distance Formula**

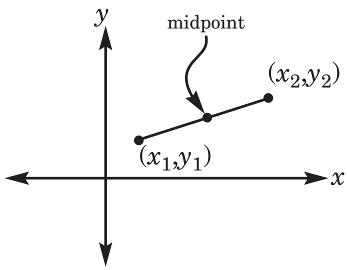
$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

**Slope Formula**

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

**Midpoint Formula**

$$(x,y) = \left(\frac{x_1 - x_2}{2}, \frac{y_1 - y_2}{2}\right)$$




When developing a coordinate geometry proof:

1 Draw and label the graph.
2 State the formulas you are using.
3 Show ALL work.
4 Have a concluding sentence stating what you have proven and why it is true.

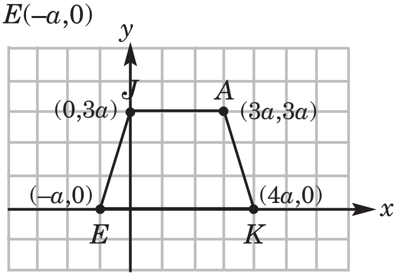
## Example

Quadrilateral *JAKE* has coordinates $J(0,3a)$, $A(3a,3a)$,$K(4a,0)$ and $E(–a,0)$. Prove by coordinate geometry that quadrilateral *JAKE* is an isosceles trapezoid.

Note: Do not let the "$a$" confuse you. It is just a multiplier. Remember $–a$ is the same as $–1a$.

## Plan

The word trapezoid, by definition, tells you that you are looking for a figure with ONLY ONE set of parallel sides. Lines are parallel when they have the same slope. You will use the Slope Formula. The word isosceles indicates the two sides will be the same length. You will use the distance formula to prove that.

| | **Statements** | | **Reasons** |
|---|---|---|---|
| 1 | $J(0,3a)$, $A(3a,3a)$, $K(4a,0)$, and $E(-a,0)$ | 1 | Given |
| 2 |  | 2 | Draw a neat, labeled graph of the problem. |
| 3 | $m = \frac{y_2 - y_1}{x_2 - x_1}$ or $m = \frac{\text{rise}}{\text{run}}$ | 3 | Use the Slope Formula to determine if the lines are parallel. |
| 4 | slope $\overline{JA} = \frac{3a - 3a}{3a - 0} = \frac{0}{3a} = 0$<br>$\overline{EK} = \frac{0 - 0}{4a - (-a)} = \frac{0}{5a} = 0$<br>$\therefore \overline{JA} \parallel \overline{EK}$ same slope<br>$\overline{EJ} = \frac{3a - 0}{0 - (-a)} = \frac{3a}{a} = 3$<br>$\overline{AK} = \frac{3a - 0}{3a - 4a} = \frac{3a}{-a} = -3$<br>$\therefore \overline{EJ}$ and $\overline{AK}$ are non parallel sides of the trapezoid. | 4 | Show ALL work! You are looking for ONE set of parallel sides and ONE set of non-parallel sides. Be sure to state the connection between the slopes and the sides being parallel or non-parallel. |
| 5 | *JAKE* is a trapezoid since it is a quadrilateral with only one set of parallel sides. | 5 | Definition of a trapezoid. |
| 6 | $\text{d} = \sqrt{(x_2 - x_1)^2 = (y_2 - y_1)^2}$ | 6 | Use the Distance Formula to determine if the non-parallel sides are of equal length. |
| 7 | length $\overline{EJ}$<br>$= \sqrt{(0-(-a))^2 + (3a-0)^2}$<br>$= \sqrt{a^2 + 9a^2} = a\sqrt{10}$<br>$\overline{AK} = \sqrt{(3a-4a)^2 + (3a-0)^2}$<br>$= \sqrt{a^2 + 9a^2} = a\sqrt{10}$<br>Therefore, $\overline{AK} \cong \overline{EJ}$ | 7 | The length of each non-parallel leg is equal. |
| 8 | Quadrilateral *JAKE* is an isosceles trapezoid. | 8 | Definition of isosceles – two sides of equal length. |

## Part 1 – Practice Questions 1A

1 Which statements could be used to prove that $\Delta ABC$ and $\Delta A'B'C'$ are congruent?

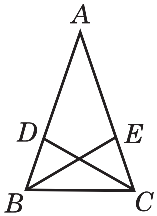
(1) $\overline{AB} \cong \overline{A'B'}$, $\overline{BC} \cong \overline{B'C'}$, and $\angle A \cong \angle A'$

(2) $\overline{AB} \cong \overline{A'B'}$, $\angle A \cong \angle A'$, and $\angle C \cong \angle C'$

(3) $\angle A \cong \angle A'$, $\angle B \cong \angle B'$, and $\angle C \cong \angle C'$

(4) $\angle A \cong \angle A'$, $\overline{AC} \cong \overline{A'C'}$, and $\overline{BC} \cong \overline{B'C'}$

2 In the accompanying diagram of $\Delta ABC$, $\overline{AB} \cong \overline{AC}$, $\overline{BD} = \frac{1}{3}\overline{BA}$, and $\overline{CE} = \frac{1}{3}\overline{CA}$.



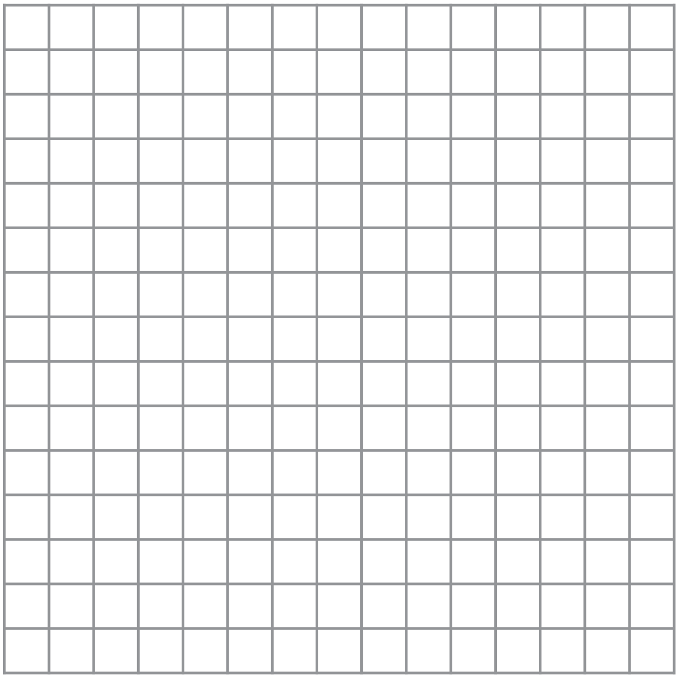

Triangle *EBC* can be proved congruent to triangle *DCB* by

(1) SAS ≅ SAS
(2) ASA ≅ ASA
(3) SSS ≅ SSS
(4) HL ≅ HL

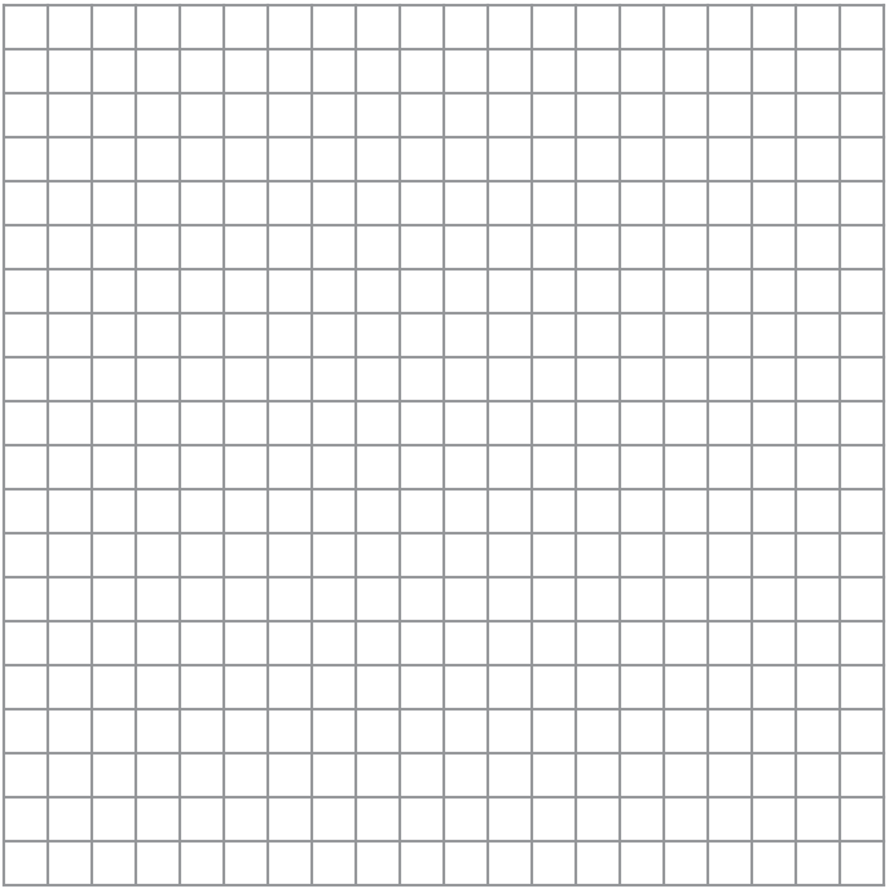
## Part 2 – Practice Questions 1A

1 Given: $A(1,6)$, $B(7,9)$, $C(13,6)$, and $D(3,1)$

Prove: *ABCD* is a trapezoid. [*The use of the accompanying grid is optional*.]

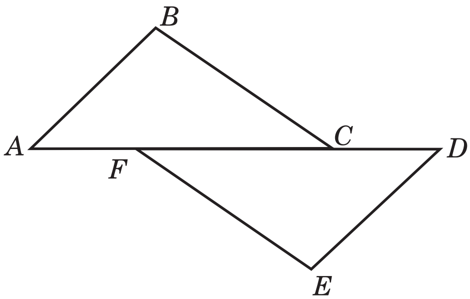


2 The coordinates of quadrilateral $ABCD$ are $A(-1,-5)$, $B(8,2)$, $C(11,13)$, and $D(2,6)$. Using coordinate geometry, prove that quadrilateral $ABCD$ is a rhombus. [*The use of the accompanying grid is optional*.]



# PART 3 – PRACTICE QUESTION 1A

1 Complete the partial proof below for the accompanying diagram by providing reasons for steps 3, 6, 8, and 9.




Given: $\overline{AFCD}$

$\overline{AB} \perp \overline{BC}$

$\overline{DE} \perp \overline{EF}$

$\overline{BC} \parallel \overline{FE}$

$\overline{AB} \cong \overline{DE}$

Prove: $\overline{AC} \cong \overline{DF}$

| Statements | Reasons |
|---|---|
| 1 $\overline{AFCD}$ | 1 Given |
| 2 $\overline{AB} \perp \overline{BC}$, $\overline{DE} \perp \overline{EF}$ | 2 Given |
| 3 $\angle B$ and $\angle E$ are right angles. | 3 ______________________ ______________________ |
| 4 $\angle B \cong \angle E$ | 4 All right angles are congruent. |
| 5 $\overline{BC} \parallel \overline{FE}$ | 5 Given |
| 6 $\angle BCA \cong \angle EFD$ | 6 ______________________ ______________________ |
| 7 $\overline{AB} \cong \overline{DE}$ | 7 Given |
| 8 $\Delta ABC \cong \Delta DEF$ | 8 ______________________ ______________________ |
| 9 $\overline{AC} \cong \overline{DF}$ | 9 ______________________ ______________________ |

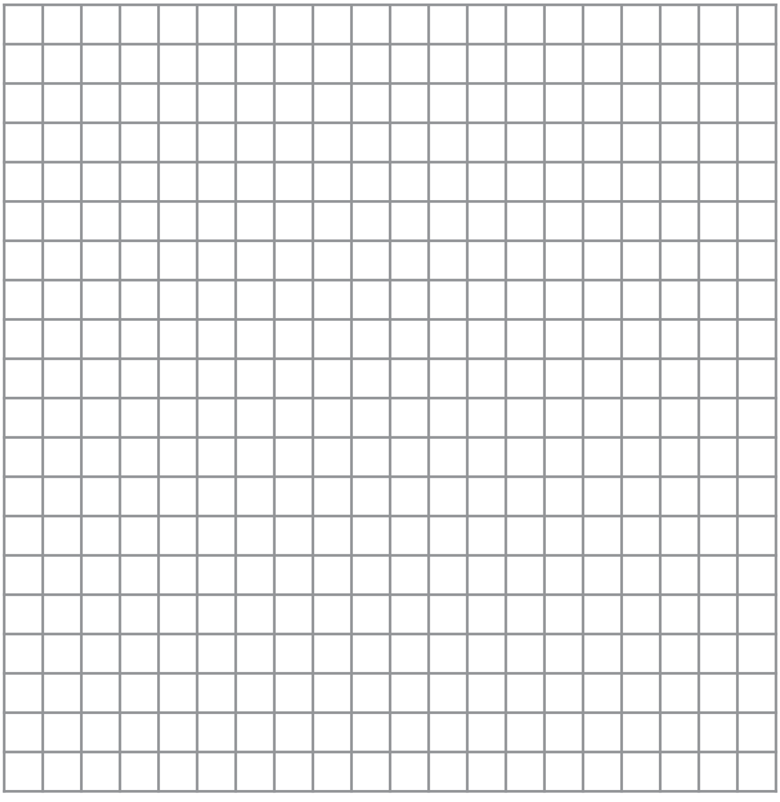
## Part 4 – Practice Question 1A

1 Prove that the diagonals of a parallelogram bisect each other.

## Classroom Activity 1A

Quadrilateral *MIKE* has coordinates:
*M*(0,11)
*I*(4,11)
*K*(8,–1)
*E*(–3,–1)

Prove by coordinate geometry that MIKE is an isosceles trapezoid.
[*The use of the accompanying grid is optional*.]



**Section 1B – PERFORMANCE INDICATOR 1B**
Construct indirect proofs.

- Euclidean indirect proofs.

# Indirect Proof (Proof By Contradiction)

When trying to prove a statement is true, it may be helpful to examine what happens if this statement was not true. This method is the basis of the Indirect Proof or Proof by Contradiction.

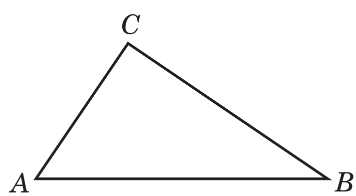
## Indirect Proof

Assume what you need to prove is false, and then show that something contradictory happens. Look for the word "not" or the presence of a not equal sign. These usually indicate the need for an indirect proof.

### Steps In An Indirect Proof

1 Assume temporarily that the conclusion is not true.
2 Reason logically until you reach a contradiction of a known fact.
3 State that the temporary assumption must be false and that the conclusion must be true.

### Example Of An Indirect Proof



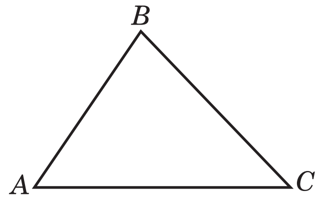

**Given** $\Delta ABC$ with $m\angle A > m\angle B$

**Prove** $BC > AC$

| Statements | Reasons |
|---|---|
| 1 $BC = AC$ | 1 Assumption |
| 2 If $BC = AC$, then $\Delta ABC$ is isosceles. | 2 Defination |
| 3 If isosceles, $\angle A \cong \angle B$ | 3 Defination |
| 4 $m\angle A > m\angle B$ | 4 Contradiction, so $BC \neq AC$ |
| 5 $BC < AC$ | 5 Assumption |
| 6 If $BC > AC$, it is possible to construct a point $D$ on $\overline{CA}$ such that $\overline{CD} \cong \overline{CB}$ | 6 Segment Duplication |
| 7 Contruct $\overline{DB}$ | 7 Point $D$ is not on line segment $\overline{CA}$. Contradiction, so $BC$ is not less than $AC$. |
| 8 Therefore, $BC > AC$ | 8 Since we proved that $BC$ cannot be equal to or less than $AC$, it must be greater than $AC$. |

## PRACTICE QUESTIONS 1B

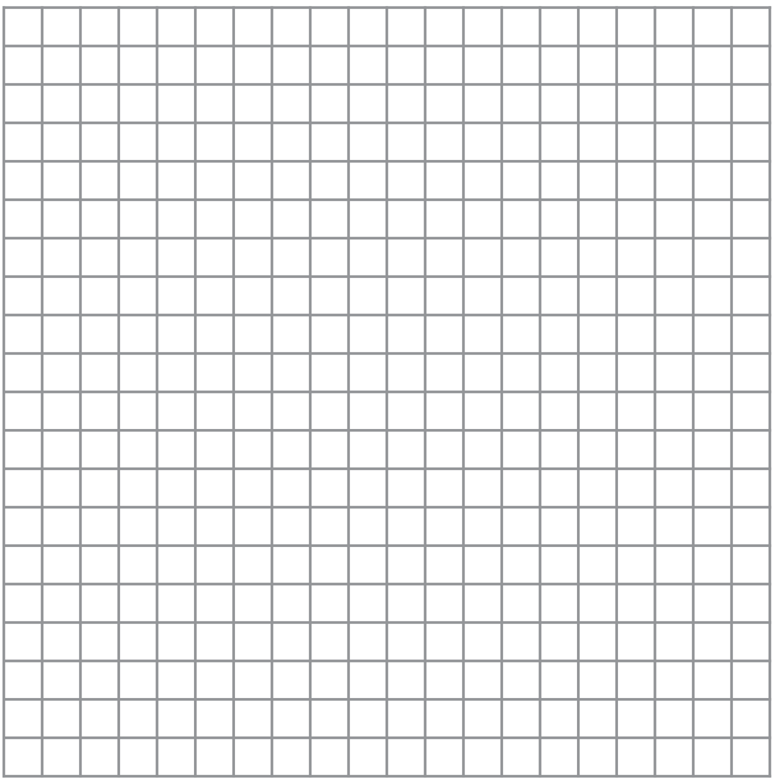
1 In the accompanying diagram, $\Delta ABC$ is *not* isosceles. Prove that if altitude $\overline{BD}$ were drawn, it would *not* bisect $\overline{AC}$.




2 Quadrilateral *KATE* has vertices $K(1,5)$, $A(4,7)$, $T(7,3)$, and $E(1,-1)$.

*a*) Prove that *KATE* is a trapezoid. [*The use of the accompanying grid is optional*.]

*b*) Prove that *KATE* is *not* an isosceles trapezoid.



## Classroom Activity 1B

For over 50 years, Dorothy, the Tin Man, the Scarecrow, and the Lion have been following the yellow brick road in the *Wizard of Oz*. In the story, the scarecrow sings "I wish I had a brain" and goes off with Dorothy to the land of Oz in search of the Wizard who can hopefully satisfy this wish. As everyone knows, there really is no Wizard, but only a man pulling levers behind a curtain. Being a clever and kind-hearted man, the ersatz wizard explains to the Scarecrow that he has had a brain all along but is only lacking a diploma to prove his intelligence. The Wizard then proceeds to bestow an honorary degree, with appropriate diploma, upon the Scarecrow. To demonstrate his newly discovered intelligence, the Scarecrow quotes the following theorem:

The sum of the square roots of any two sides of an isosceles triangle is equal to the square root of the remaining side.

Prove or disprove this theorem.